

ASSISTANT RESEARCH OFFICER (PAPER-II) (S.A.T.)

Time Allowed: 03 Hours

Maximum Marks: 120

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

1. There are **EIGHT** questions in **TWO** Parts in each subject, i.e. Economics, Mathematics and Statistics.
2. The candidate has to attempt **(06) SIX** questions in all either in English or Hindi by choosing at least **(03) THREE** questions from each part of the concerned subject.
3. All questions carry equal marks. Each question will consist of 04 sub parts having 05 marks and word limit will be 150 words for each sub-part.
4. The candidate has to choose one subject as per his/ her choice, i.e. **Economics, Mathematics and Statistics (subject code: 0001, 0002 and 0003, respectively)**. The subject opted by the candidate is required to be encoded/ written on the outer OMR portion of the Answer Booklet in the prescribed columns titled as 'SUBJECT/ PAPER' and 'SUBJECT CODE'.
5. Write answers in legible handwriting. Illustrate your answers with suitable sketches, diagrams and figures, wherever considered necessary.
6. Each part of the question must be answered in sequence and in the same continuation.
7. Attempts of the questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in answer booklet must be clearly struck off.
8. In case of bilingual paper(s), if there is any difference in English and Hindi version of the question, the English version shall be treated as correct and final.

IMPORTANT NOTE: ANSWER ANY (03) THREE QUESTIONS FROM EACH PART.

Economics (Subject code: 0001)

PART-I (60 Marks)

Q. No. 1: Describe the following:-

- (1) State and explain the Law of Diminishing Marginal Utility.
- (2) What are the main causes of income inequalities in India? Discuss.
- (3) Define Adding-up Theorem. Show that when factors of production are paid equal to their marginal products, the total product would be just exhausted.
- (4) How New Welfare Economics is different from Pareto Optimality? Elucidate your answer.

- (1) ह्रासमान सीमान्त उपयोगिता के नियम को बताइये तथा व्याख्या कीजिए।
- (2) भारत में आय की असमानताओं के प्रमुख कारण क्या हैं? विवेचना कीजिए।
- (3) योग प्रमेय को परिभाषित कीजिए। यह दिखाइये कि जब उत्पादन के साधनों को उनके सीमान्त उत्पाद के बराबर भुगतान किया जाता है तो कुल उत्पाद पूरी तरह से समाप्त हो जाता है।
- (4) किस प्रकार नया कल्याणवादी अर्थशास्त्र परेटो अनुकूलतम से भिन्न है? अपने उत्तर को समझाइये।

Q. No. 2: Describe the following:-

- (1) Differentiate between matrices and determinants. Write a note on their uses in Economics.
- (2) A firm produces X units of output per week at a total cost of ₹ $(\frac{1}{3}X^3 - X^2 + 5X + 3)$. Find the output at which marginal variable cost and average variable cost are same.
- (3) Critically discuss the marginal productivity theory of wages.
- (4) Present a critical review of social security measures in India.

- (1) आव्यूहिकी तथा सारणिकों में भेद कीजिए। अर्थशास्त्र में उनके उपयोगों पर एक टिप्पणी लिखिए।
- (2) एक फर्म वस्तु की X इकाइयों का उत्पादन प्रति सप्ताह कुल लागत ₹ $(\frac{1}{3} X^3 - X^2 + 5 X + 3)$ पर करता है। उत्पाद की वह मात्रा बताइये जिस पर सीमान्त परिवर्तनीय लागत तथा औसत परिवर्तनीय लागत समान हो।
- (3) मजदूरी के सीमान्त उत्पादकता सिद्धान्त की आलोचनात्मक व्याख्या कीजिए।
- (4) भारत में सामाजिक सुरक्षा उपायों की समालोचनात्मक समीक्षा प्रस्तुत कीजिए।

Q. No. 3: Describe the following:-

- (1) Explain the demand pull inflation and cost push inflation with suitable diagrams.
- (2) Why is there an 'upper and lower' turning point in a trade cycle? Explain.
- (3) Critically examine the Purchasing Power Parity theory of foreign exchange rate.
- (4) How do open market operations regulate effective demand in an economy?

- (1) मांग जन्य मुद्रास्फीति तथा लागत जन्य मुद्रास्फीति को उचित रेखाचित्रों के माध्यम से समझाइये ।
- (2) किसी व्यापार चक्र में 'ऊपर ' तथा ' नीचे ' मोड़ - बिन्दु क्यों होते हैं?
- (3) विदेशी विनिमय दर के क्रयशक्ति समता सिद्धान्त का आलोचनात्मक परीक्षण कीजिए।
- (4) किसी अर्थव्यवस्था में खुले बाजार की क्रियायें किस प्रकार प्रभावी मांग को नियमित करती हैं?

Q. No. 4: Describe the following:-

- (1) Mention the main characteristics of a normal frequency distribution.
- (2) What is an Ideal Index Number? Compute Fisher's Index Number from the following data:

Commodity	2020		2023	
	Price	Quantity	Price	Quantity
A	12	10	15	12
B	15	7	20	5
C	24	5	20	9
D	5	16	5	14

- (3) 'The essence of the physiocrats lay in their concept of natural order.' Discuss.
- (4) Give a brief account of the instruments of monetary policy.

(1) प्रसामान्य आवृत्ति बंटन की प्रमुख विशेषताओं का उल्लेख कीजिए ।

(2) एक आदर्श सूचकांक क्या होता है ? निम्न समकों से फिशर के सूचकांक की गणना कीजिए:

वस्तु	2020		2023	
	कीमत	मात्रा	कीमत	मात्रा
A	12	10	15	12
B	15	7	20	5
C	24	5	20	9
D	5	16	5	14

- (3) 'प्रकृतिवादियों की मूलधारा उनके प्राकृतिक व्यवहार की अवधारणा में निहित है।' व्याख्या कीजिए ।
- (4) मौद्रिक नीति के उपकरणों का संक्षिप्त विवरण प्रस्तुत कीजिए ।

PART-II (60 marks)

Q. No. 5: Describe the following:-

- (1) State and explain the reasons for declining contribution of agricultural sector in national income of India.
- (2) Discuss the problem of rural indebtedness. What remedial measures would you suggest?
- (3) Briefly discuss the status of women in formal and informal sector.
- (4) Mention the qualities of a good schedule/questionnaire.

- (1) भारत की राष्ट्रीय आय में कृषि क्षेत्र के घटते हुए योगदान के कारणों को बताइये तथा समझाइये।
- (2) ग्रामीण ऋणग्रस्तता की समस्या की विवेचना कीजिए। इसमें सुधार हेतु आप क्या सुझाव देंगे?
- (3) औपचारिक तथा अनौपचारिक क्षेत्र में महिलाओं की स्थिति की संक्षेप में विवेचना कीजिए।
- (4) एक अच्छी अनुसूची / प्रश्नावली की विशेषताओं का उल्लेख कीजिए।

Q.No.6: Describe the following:-

- (1) What are the methods of data collection? Why sampling method is preferred over census method? Elucidate your answer.
- (2) What is understood by a research design? Specify the subject matter of a research design.
- (3) Evaluate the Wiseman- Peacock Hypothesis with reference to public expenditure. Is it relevant in Indian context?
- (4) Distinguish between internal and external public debt. Describe the various methods of repayment of internal public debt.

- (1) समंक - संग्रहण की विधियों को बताइये। प्रतिदर्श विधि को जनगणना विधि की तुलना में क्यों प्राथमिकता दी जाती है ? अपने उत्तर को स्पष्ट कीजिए।
- (2) शोध प्रकल्प से आप क्या समझते हैं? एक शोध प्रकल्प की विषय - वस्तु को चिन्हित कीजिए।
- (3) सार्वजनिक व्यय के संदर्भ में वाइज़मेन - पीकॉक उपकल्पना का मूल्यांकन कीजिए। क्या यह भारतीय संदर्भ में प्रासंगिक है?
- (4) आंतरिक तथा बाह्य सार्वजनिक ऋण में भेद कीजिए। आंतरिक सार्वजनिक ऋण के पुनर्भुगतान की विभिन्न विधियों का वर्णन कीजिए।

Q. No. 7: Describe the following:-

- (1) Explain Weber's theory of industrial location.
- (2) Discuss the role of MSMEs in the Indian economy.

(3) Explain the Optimum Theory of Population. Give views of Dalton and Robbins in this context.

Or

Define autocorrelation. What factors cause autocorrelation ?

(4) Describe in brief Everett Lee's theory of migration.

Or

Mention the kinds of econometric models.

(1) बेबर के औद्योगिक स्थानीकरण के सिद्धान्त को समझाइये।

(2) भारतीय अर्थव्यवस्था में एम एस एम ई की भूमिका की विवेचना कीजिए।

(3) जनसंख्या के अनुकूलतम सिद्धान्त को समझाइये। इस संदर्भ में , डाल्टन तथा रॉबिन्स के विचारों को बताइये ।

या

स्वसहसंबंध को परिभाषित कीजिए । किन कारकों के कारण स्वसहसंबंध होता है?

(4) एवेरेट्ट ली के देशान्तरण सिद्धान्त की संक्षेप में व्याख्या कीजिए ।

या

अर्थमितीय मॉडलों के प्रकारों का उल्लेख कीजिए।

Q. No.8: Describe the following:-

(1) Examine critically the Rostow's Theory of Growth.

(2) Do you agree that rural unemployment has increased in India in the recent years? Substantiate your answer and suggest measures to deal with this problem.

(3) Discuss the pros and cons of Swadeshi movement vis- a- vis globalization.

(4) Highlight the importance of tourism for the economy of Himachal Pradesh.

(1) रोस्टोव के संवृद्धि सिद्धान्त का आलोचनात्मक परीक्षण कीजिए ।

(2) क्या आप इस विचार से सहमत हैं कि अभी हाल के वर्षों में भारत में ग्रामीण बेरोजगारी में वृद्धि हुई है? अपने उत्तर के पक्ष में प्रमाण दीजिए तथा इस समस्या से निपटने के लिए उपाय सुझाइये ।

(3) स्वदेशी आन्दोलन बनाम वैश्वीकरण के पक्ष -विपक्ष में तर्क दीजिए ।

(4) हिमाचल प्रदेश की अर्थव्यवस्था के लिए पर्यटन के महत्व पर प्रकाश डालिए।

Mathematics (Subject Code: 0002)

PART-I (60 Marks)

1. (i) Let (X, d) be a metric space. Prove that each open sphere in X is an open set.
- (ii) Let (Y, d_Y) be a subspace of a metric space (X, d) . Prove that Y is complete $\Rightarrow Y$ is closed.
- (iii) Define Bolzano-Weierstrass property and prove that a compact metric space has the Bolzano-Weierstrass property.
- (iv) If $T: V \rightarrow \tilde{V}$ be a linear operator from an n -dimensional linear space V into a linear space \tilde{V} , prove that

$$\dim \aleph(T) + \dim \mathbb{R}(T) = \dim V.$$

2. (i) Find the asymptotes of the curve

$$x^3 + y^3 = 3axy.$$

- (ii) If $u = \sec^{-1} \left(\frac{x^3+y^3}{x+y} \right)$, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u.$$

- (iii) Prove that the product of two functions of bounded variation is also of bounded variation.
 - (iv) If $\{f_n\}$ is a sequence of continuous functions on an interval $[a, b]$ and if $f_n \rightarrow f$ uniformly on $[a, b]$, then prove that f is continuous on $[a, b]$.
3. (i) The number of bacteria in a certain culture grows at a rate proportional to the number present. Show that for arbitrary constants A and k , the number of bacteria at time t is equal to Ae^{kt} .

- (ii) Under which operation, the set

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Forms a group. Verify?

- (iii) Prove that any field is an integral domain.
- (iv) If R is a commutative ring with unit element and M is an ideal of R , prove that M is a maximal ideal of R if R/M is a field.
4. (i) Find the maximum value of $P = 8x + 3y$, subject to the constraints

$$x \geq 0, y \geq 0, x + y \leq 3, 4x + y \leq 6$$

- (ii) Show that an analytic function in a domain with its derivative zero for each point of the domain is constant.
- (iii) Show that the transformation $w = \frac{1}{z}$ maps a horizontal line $y = c_2 (c_2 \neq 0)$ onto the circle

$$u^2 + \left(v + \frac{1}{2c_2}\right)^2 = \left(\frac{1}{2c_2}\right)^2,$$

Where $w = u + iv$ and $z = x + iy$.

- (iv) Find the centre of gravity of three equal particles placed at the vertices of a triangle.

PART-II (60 Marks)

5. (i) Six boys and six girls sit together in a row randomly. Find the probability that the six girls sit together.
- (ii) If $2x + y = 7$ and $x + 2y = 7$ are the two regression lines, respectively, then find the correlation coefficient between X and Y .
- (iii) Find the value of $\int_0^1 \frac{1}{1+x} dx$ by trapezoidal rule taking $n = 2$.

- (iv) Find the Fourier transform of the function defined by

$$\chi_{[-a,a]}(t) = \begin{cases} 1, & |t| \leq a \\ 0, & |t| > a \end{cases}, a > 0$$

6. (i) Let X be any uncountable set and
 $\tau = \{G : G = \phi \text{ or } G \subset X \text{ such that } X - G \text{ is countable}\}$

Show that τ is a topology on X .

- (ii) Let (X, τ) be a topological space. Prove that X is second countable space $\Rightarrow X$ is first countable space.
- (iii) Let (X, τ) be a compact space, (Y, τ') a Hausdorff space and

$$f: X \xrightarrow[\text{onto}]{1-1} Y$$

be a continuous function. Prove that f is a homeomorphism.

- (iv) Let (X, τ) be a topological space. If D is a component of X , then prove that D is closed.
7. (i) Let X and Y be normed spaces over the field \mathbb{K} and $T: X \rightarrow Y$ a linear operator. Prove that T is continuous if and only if T is bounded.
- (ii) Prove that all norms on a finite dimensional linear space are equivalent.
- (iii) Prove that normed space is isometrically isomorphic to a dense subspace of a Banach space.
- (iv) If x and y are any two vectors in an inner space X , then prove that
- $$|\langle x, y \rangle| \leq \|x\| \|y\|.$$
8. (i) Prove that the intersection and difference of two measurable sets are measurable.
- (ii) Prove that each step function is a simple function.
- (iii) If a sequence $\{f_n\}$ converges in measure to f , then prove that the limit function f is unique a.e.
- (iv) Let $\{f_n\}$ be an increasing sequence of non-negative measurable functions and let $f = \lim_{n \rightarrow \infty} f_n$. Prove that

$$\int f = \lim_{n \rightarrow \infty} \int f_n.$$

Statistics (Subject Code: 0003)

Note: Use of Calculator, log tables and statistical tables is allowed.

PART– I (60 marks)

Q. No. 1. Describe the following:-

(1). Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

(2). Let T be a linear transformation from R^3 into R^2 , where $T(X) = AX, A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $X = (x, y, z)'$. Find $range(T)$, $rank(T)$ and $Kernel(T)$ with its dimension.

(3). Evaluate the following integral

$$\int_0^1 \frac{dx}{3+2x}$$

using trapezoidal rule with $n = 4$

(4). Find the values of α and β , so that following system of linear equations has infinite number of solutions.

$$x + y + 2z = 3, \quad x + \alpha z = \beta, \quad y + 3z = 3.$$

Q. No. 2. Describe the following:-

(1). From the data relating to the yield of dry bark (X_1), height (X_2) and girth (X_3) for 18 plants, the following correlation coefficient were obtained:

$$r_{12} = 0.75, \quad r_{13} = 0.70 \text{ and } r_{23} = 0.50.$$

Find the partial correlation coefficient $r_{12.3}$ and multiple correlation coefficient $R_{1.23}$.

(2). For a binomial distribution consisting of $n=5$ independent trials with the probability of success as p , the probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

(3). Show that mean of Cauchy distribution does not exist.

(4). If X_1 and X_2 are independent random variates on Uniform $U(0, 1)$, find the distribution of X_1X_2 .

Q. No. 3. Describe the following:-

(1). Let Y be a random variable with moment generating function $M_y(t) = e^{3t}(1-5t)^{-1}$. Obtain the moment generating function of $X = 2Y - 6$.

- (2). Show that for t -distribution with n degree of freedom, mean deviation about the mean is given by:

$$\sqrt{n}\Gamma\left[\frac{n-1}{2}\right]/\sqrt{\pi}\Gamma\left(\frac{n}{2}\right)$$

- (3). Following table gives the sales of a leading departmental store. Fit a straight-line trend by method of least squares and obtain the trend values. Estimate the sales of the store for the year 2006 and 2007.

Year:	1995	1997	1998	1999	2000	2001	2004
Sales (Crores Rs.):	77	88	94	85	91	98	90

- (4). On a certain date the Ministry of Labour retail price index was 204.6. The percentage increase in price over some base period were: Rent 65, Clothing 220, Fuel and Light 110, Miscellaneous 125. What was the percentage increase in food, given that the weights of the different items were: Food 60, Rent 16, Clothing 12, Fuel and Light 8 and Miscellaneous 4?

Q. No. 4. Describe the following:-

- (1). Prove that the Maximum likelihood estimate of the parameter θ of a population having density function:

$$f(x) = \frac{2}{\theta^2}(\theta - x) \quad 0 < x < \theta,$$

for a sample of unit size is $2x$, x being sample value. Show that the estimate is biased.

- (2). Let X_1, X_2, \dots, X_n be a random sample from uniform $U[0, \theta]$, $\theta > 0$ population. Show that $T = \max_{1 \leq i \leq n} X_i = X_{(n)}$ is complete sufficient statistics for θ .
- (3). If $x \geq 1$ is the critical region for testing $H_0: \theta = 2$ against $H_1: \theta = 1$, on the basis of single observation from the population $f(x, \theta) = \theta e^{-\theta x}$; $0 \leq x < \infty$. Obtain the values of type I and type II errors.
- (4). In stratified random sampling with two strata, a sampler wants to take $n_1 = n_2$ instead of the values $(n_1)_N$ and $(n_2)_N$ given by the Neyman allocation. Let $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_N$ denote the variances given by $n_1 = n_2$ and Neyman allocation respectively. Show that the fractional increase in variance is:

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_N}{V(\bar{y}_{st})_N} = \left(\frac{r-1}{r+1}\right)^2$$

where $r = (n_1)_N / (n_2)_N$.

PART-II (60 marks)

Q. No. 5. Describe the following:-

- (1). Show that the function

$$f(z) = \frac{z^2 + 2z + 1}{z^2 + 5z + 2}$$

has a pole of order 2 at $z = \infty$.

- (2). Using the method of differences, construct a Balanced Incomplete Block Design (BIBD) for $v = 7$ treatments arranged in blocks of size $k = 3$.
- (3). Discuss the testing of hypothesis and estimation of confidence intervals in case of a simple linear regression model.
- (4). What is meant by autocorrelation? Discuss the theoretical and practical consequences of autocorrelation in detail.

Q. No. 6. Describe the following:-

- (1). For a 2^4 -factorial experiment, the key block is given as (1), ac, cd, ad, b, abc, bcd and abd. Identify the confounded effect and obtain the other block.
- (2). Consider the model $E(y_1) = 2\beta_1 + \beta_2$, $E(y_2) = \beta_1 - \beta_2$, and $E(y_3) = \beta_1 + \alpha\beta_2$ with $V(y_i) = \sigma^2; i = 1, 2, 3$ and $Cov(y_i, y_j) = 0$ for all $i \neq j$. Obtain the value of α so that the best linear unbiased estimators of β_1 and β_2 are uncorrelated.
- (3). How do you set the control limits for R - chart in statistical quality control? Explain briefly.
- (4). Find the stationary distribution of Markov chain with the transition matrix $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$.

Q. No. 7. Describe the following:-

- (1). Define a consumer and consumer's risk. How is consumer's risk different from producer's risk?
- (2). Discuss the simplex method of linear programming problem with an example.
- (3). Derive expression for the reliability of a test at length ' k ' in terms of its reliability at length ' h '. Suppose that a test has reliability coefficient of 0.60. Find (i) its reliability when its length is doubled and (ii) the length to which it must be extended so as to have reliability coefficient of 0.90.
- (4). Using Wolfowitz Run test examine whether the two sample are coming from the same population.

X: 75.1, 78.7, 75, 75.4, 74.8, 80.3, 76.8, 83.8, 73.2, 76.2, 78.1, 79.4, 77.3, 77.8, 79.2

Y: 71.8, 75.9, 75.2, 71.4, 70.2, 73.3, 77.9, 73.4, 78.4, 79.1, 73.8, 75.3, 75.7

Q. No. 8. Describe the following:-

- (1). Consider an experiment involving six treatments in four randomized blocks. The sum of squares are given as: Sum of squares due to treatments (SST) = 901.19, Total sum of squares (TSS) = 1350.25, Sum of squares due to blocks (SSB) = 219.43. Construct the ANOVA table of the experiment.

- (2). The data matrix for a random sample of size $n = 3$ from a bivariate normal population is

$$X = \begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}.$$

Evaluate the observed T^2 for $\mu'_0 = [9, 5]$. What is the sampling distribution of T^2 ?

- (3). Find a generalized inverse G of the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ and show that $AGA=A$.

- (4). What is meant by local control and in what way does it increase the efficiency of an experimental design?