Syllabus for descriptive type Subject Aptitude Test (SAT) for the recruitment to post of Assistant Research Officer, Class-II (Non-Gazetted) in the Department of Planning, H.P. The SAT paper shall have two parts i.e. Part-I and Part-II and cover the following topics of Master of Science (Mathematics) level.

PART-I (60 MARKS)

1. <u>LINEAR ALGEBRA:-</u>

Vector spaces over R and C, linear dependence and independence, subspaces, bases, dimension, deletion and replacement theorem, Linear transformations, rank and nullity, matrix of a linear transformation, Algebra of Matrices, Row and column reduction, Echelon form, congruence's and similarity, Rank of a matrix, Inverse of a matrix, Solution of system of linear equations, Euclidean space, Gram Schmidt orthogonalization, Symmetric, skew symmetric, Hermitian, skew Hermitian, orthogonal and unitary matrices and their eigenvalues. Quadratic forms, diagonalization of symmetric matrices.

The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

2. <u>REAL ANALYSIS:-</u>
Real number system as an ordered field with least upper bound property, Functions of a real variable, limits, continuity. Intermediate value theorem, Differentiability, Rolle's theorem, mean-value theorem, L'Hospital's rule, Maxima and minima; asymptotes; envelopes, Higher order differentiation, Leibnitz' formula, Taylor's theorem with remainders.

Sequences and Series and their convergence, absolute and conditional convergence of series of real and complex numbers, Sequences and series of functions, Uniform convergence and power series.

Open sets, limit points, closed sets. Bolzano-Weierstrass theorem, Compact sets, Nested interval theorem, Heine Borel theorem, Uniform continuity of functions. .

3. <u>POLYNOMIAL EQUATION AND BASIC INTEGRATION THEORY:-</u>

Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Applications to solution of equations; De Moivre's theorem for rational indices, the nth roots of unity and symmetries of the solutions. Transformation of equations, Cardon's method of solving cubic and Descartes' method of solving biquadratic equations, Elementary symmetric functions and symmetric functions of the roots of an equation; Newton's theorem on sums of the like powers of the roots; Transformation of equations by symmetric functions.

Darboux integrability; Riemann's integrability, Riemann-Stieltjes integral. Riemann integrability of monotone functions, continuous functions, piecewise continuous functions and piecewise monotone functions; Fundamental Theorems of Calculus (I and II), Methods of integration; Volume by slicing and cylindrical shells, Length of a curve in the plane and the area of surfaces of revolution, Improper integrals of Type-I, Type-II and mixed type, Convergence of improper integrals, the beta and gamma functions and their properties.

4. ORDINARY DIFFERNTIAL EQUATIONS:-

Implicit, general and singular solutions for the first order ordinary differential equation; Bernoulli's equation, Initial value problems, Reducible second order differential equations; Applications of first order differential equations to Newton's law of cooling, exponential growth and decay problems. General solution of homogenous equation of second order, Wronskian and its properties, Linear homogeneous and non homogeneous equations of higher order with constant coefficients, Method of variation of parameters, Method of undetermined coefficients, Two-point boundary value problems, Cauchy- Euler's equation, System of linear differential equations, Application of second order differential equation; Various models and ther analysis;

Well posed problems, Existence, Solution of simultaneous differential equations of first order and ODEs of higher order, Sturm separation and comparison theorems, Homogeneous linear systems, Non-homogeneous linear systems, Two point boundary value problems, Green's function, Stability of autonomous system of differential equations, Critical point of an autonomous system and their classification as stable, Asymptotically stable, Strictly stable and unstable, Stability of linear systems, Linear plane autonomous systems, Perturbed systems, Method of Lyapunov for nonlinear systems.

Nonlinear Differential Equations: Phase Plane, Paths, and Critical Points, Critical Points and paths of Linear Systems, Critical Points and Paths of Nonlinear Systems. Limit Cycles and Periodic Solutions. The Method of Kryloff and Bogoliuboff.

5. GROUPS, RINGS and MODULES:-

Groups, examples of Groups, classification of subgroups of cyclic groups, permutation groups and group of symmetries, alternating groups, Cosets, Lagrange's theorem and various consequences, Normal subgroups, Factor groups, Cauchy's theorem for finite Abelian groups, Group homomorphism and isomorphism, Cayley's theorem; Isomorphism theorems for groups, Automorphism, Inner automorphism, Automorphism groups of cyclic groups, Applications of factor groups to automorphism groups.

Rings, various examples of Rings, Subrings, Integral domain, Ideals, Factor rings, Prime ideals and maximal ideals, Principal ideal domains. Ring homomorphisms; Isomorphism theorems for rings; the field of quotients; Polynomial rings over commutative rings, Eisenstein's criterion, Unique factorization in $\mathbb{Z}[x]$, Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.

Modules, sub-modules and direct sums, homomorphisms and quotient modules, Completely reducible modules, Free modules.

6. COMPLEX ANALYSIS:-

Limits and continuity. Analytic functions. Polynomials and rational functions, Continuity and differentiation, Cauchy-Riemann equations and examples, Sufficient conditions for differentiability, Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions and their properties, Derivatives of functions, Definite integrals of functions; Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals; Antiderivatives; Cauchy-Goursat theorem; Cauchy integral formula and its extension with consequences; Liouville's theorem and the fundamental theorem of algebra.

Conformal mapping, length and area, Mobius transformations, Power series representation

of analytic functions, Maximum modulus theorem, Index of a closed curve, Cauchy's theorem and integral formula on open subset of complex plane, Homotopy, Homotopic version of Cauchy's theorem, Simple connectedness, Counting zeros and open mapping theorem, Goursat's theorem, Classification of singularities, Laurent series, Residue, Contour integration, Argument principle, Rouche's theorem, Maximum principles, Schwarz' lemma, residuetheorem and evaluation of definite integral.

7. FLUID DYNAMICS:-

Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Irrotational flow, Vorticity vector, Equipotential surfaces, Streamlines, pathlines and streak lines of particles, Stream tube and stream surface, Mass flux density, Conservation of mass leading to equation of continuity (Euler's form), Boundary surface, Conservation of momentum and its mathematical formulation (Euler's form), Integration of Euler's equation under different conditions, Bernoulli's equation, steady motion under conservative body forces.

Theory of irrotational motion, Kelvin's minimum energy and circulation theorems, Potential theorems, Two-dimensional flows of irrotational, incompressible fluids, Complex potential, Sources, sinks, doublets and vortices, Milne-Thomson circle theorem, Images with respect to a plane and circles, Blasius theorem.

Three-dimensional flows, Sources, sinks, doublets, Axi-symmetric flow and Stokes stream function, Butler sphere theorem, Kelvin's inversion theorem, Weiss's sphere theorem, Images with respect to a plane and sphere, Axi-symmetric flows and stream function, Motion of cylinders and spheres, Viscous flow, stress and strain analysis, Stokes hypothesis, Navier-Stokes equations of motion, Some exactly solvable problems in viscous flows, Steady flow between parallel plates, Poiseuille flow, Steady flow between concentric rotating cylinders.

8. LINER PRAGRAMMING PROBLEM:-

Linear programming problem: Standard, Canonical and matrix forms, Graphical solution; Convex and polyhedral sets, Hyper-planes, Extreme points; Basic solutions, Basic feasible solutions, Reduction of feasible solution to a basic feasible solution, Correspondence between basic feasible solutions and extreme points. Simplex method: Optimal solution, Termination criteria for optimal solution of the linear programming problem, Unique and alternate optimal solutions, Un-boundedness; Simplex algorithm and its tableau format; Artificial variables, Two-phase method, Big-M method, Formulation of dual problem; Primal
Dual relationships; Fundamental theorem of duality; Complimentary slackness.

Transportation Problem: Definition and formulation; Methods of finding initial basic feasible solutions; Northwest-corner rule. Least- cost method; Vogel's approximation method; Algorithm for solving transportation problem. Assignment Problem: Mathematical formulation and Hungarian method of solving. Game Theory: Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear programming method of solving a game.

9. INNER PRODUCT SPACES and OPERATORS:-

Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to

least squares approximation and minimal solutions to systems of linear equations, Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

Polar decomposition, Simultaneously Diagonalizable Matrices, Unitary equivalence, Schur's theorem, the eigenvalues of sum and product of commuting matrices, Normal matrices, spectral theorem for normal matrices, Simultaneously unitarily diagonalizable commuting normal matrices. Matrix norms, Examples, Operator norms, Matrix norms induced by vector norms, The spectral norm, Frobenius norm, Unitary invariant norm, The maximum column sum matrix norms, the maximum row sum matrix norm, Positive definite matrices, Definitions and properties, Characterizations, The positive semi- definite ordering, Loewner partial order, Inequalities for the positive definite matrices, Hadamard's inequality, Fischer's inequality, Minkowski's inequality.

10. <u>METRIC SPACES</u>:-

Definition, examples, sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Closed set, Limit points, Closure of a set, Cantor's theorem, Subspaces, Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity; Homeomorphism, Isometry and equivalent metrics, Contraction mapping, Banach fixed point theorem.

Compactness and Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces, Weierstrass approximation theorem.

11. <u>MECHANICS</u>:-

Coplanar force systems; Three-dimensional force systems; Moment of a force about a point and an axis, Principle of moments, Couple and couple moment, Moment of a couple about a line, Resultant of a force system, Distributed force system, Rigid-body equilibrium, Equilibrium of forces in two and three dimensions, Free-body diagrams, General equations of equilibrium, Constraints and statical determinacy, Equations of equilibrium and friction, Frictional forces on screws and flat belts; Center of gravity, Center of mass and Centroid of a body and composite bodies; Theorems of Pappus and Guldinus; Moments and products of inertia for areas, Composite areas and rigid body, Parallel-axis theorem, Moment of inertia of a rigid body about an arbitrary axis, Principal moments and principal axes of inertia, Conservative force fields, Conservation of mechanical energy, Work-energy equations, Kinetic energy and work-kinetic energy expressions based on center of mass, Moment of momentum equation for a single particle and a system of particles, Translation and rotation of rigid bodies, Chasles' theorem, General relationship between time derivatives of a vector for different references, Relationship between velocities of a particle for different references, Acceleration of particle for different references.

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction. Kinemetics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane

dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits, Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

12. NUMBER THEORY:-

Linear Diophantine equation, Prime counting function, Prime number theorem, Goldbach conjecture, Fermat and Mersenne primes, Congruence relation and its properties, Linear congruence and Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Number theoretic functions for sum and number of divisors, Multiplicative function, Mobius inversion formula, Greatest integer function. Euler's phi-function and properties, Euler's Theorem, The order of an integer modulo n, Primitive roots for primes, Composite numbers having primitive roots; Definition of quadratic residue of an odd prime, and Euler's criterion, Quadratic Reciprocity Law and Public Key Encryption, The Legendre symbol and its properties, Quadratic reciprocity, Quadratic congruencies with composite moduli; Public key encryption, RSA encryption and decryption.

Algebraic numbers, Conjugates and discriminants, Algebraic integers, Integral bases, Norms and traces, Rings of algebraic integers, Quadratic and cyclotomic fields, Trivial factorization, Factorization into irreducibles, Examples of non-unique factorization into irreducibles, Prime factorization, Euclidean domains, Euclidean quadratic fields, Consequence of unique factorization.

13. MATHEMATICAL FINANCE:-

Interest rates, Types of rates, Measuring interest rates, Zero rates, Bond pricing, Forward rates, Duration, Convexity, Exchange-traded markets and Over-the-counter markets, Derivatives, Forward contracts, Futures contracts, Options, Types of traders, Hedging, Speculation, Arbitrage, No Arbitrage principle, Short selling, Forward price for an investment asset.

Types of options, Option positions, Underlying assets, Factors affecting option prices, Bounds for option prices, Put-call parity (in case of non-dividend paying stock only), Early exercise, Trading strategies involving options (except box spreads, calendar spreads and diagonal spreads), Binomial option pricing model, Risk-neutral valuation (for European and American options on assets following binomial tree model).

Brownian motion (Wiener Process), Geometric Brownian Motion (GBM), The process for a stock price, Itô's lemma, Lognormal property of stock prices, Distribution of the rate of return, Expected return, Volatility, Estimating volatility from historical data, Derivation of the Black Scholes-Merton differential equation, Extension of risk-neutral valuation to assets following GBM (without proof), Black−Scholes formulae for European options, Hedging parameters - The Greek letters: Delta, Gamma, Theta, Rho and Vega; Delta hedging, Gamma hedging.

PART-II (60 MARKS)

14. PROBABILITY AND STATISTICS:-

Descriptive statistics, measures of location and variability; Sample spaces and events, probability axioms and properties, conditional probability, Bayes' theorem and independent events; discrete random variables and probability distributions, expected values; probability distributions: binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

Continuous random variables, probability density functions, uniform distribution, cumulative distribution functions and expected values, the normal, exponential and lognormal distributions, sampling distribution and standard error of the sample mean, central limit theorem and applications; regression line using principle of least squares, estimation using the regression lines; sample correlation coefficient and properties.

15. DISCRETE MATHEMATICS AND GRAPH THEORY:-

Cardinality of sets, Partially ordered sets, Order-isomorphisms, Duality principle, Building new ordered sets, Maps between ordered sets, Lattices as ordered sets and as algebraic structures, Lattice isomorphism; Modular Distributive and Complemented lattice, Boolean Algebras, Boolean polynomials and Boolean functions, Applications of Boolean algebras to logic, set theory and probability theory.

Graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Connected graphs, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Shortest path, Dijkstra's algorithm.

Applications of Path and Circuits: The Chinese Postman Problem, Digraphs, Bellman-Ford Algorithm, Spanning Trees, Minimum Spanning Tree Algorithms, Cut-vertices, Blocks and their Characterization, Connectivity and edge-connectivity, Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring and applications, Hall's theorem, Independent sets and covers.

16. LEBESGUE MEAUSRE:-

Lebesgue outer measure, Measurable and non-measurable sets, Regularity, Measurable functions, Littlewood's three principles, Borel and Lebesgue measurability, Non-measurable sets, Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Functions of bounded variation, Lebesgue differentiation theorem, Differentiation and integration, Absolute continuity of functions, Measures and outer measures, Measure spaces, Integration with respect to a measure, The L^p-spaces, Holder and Minkowski inequalities, Completeness of L^p-spaces, Convergence in measure, Almost uniform convergence, Egorov's theorem.

17. FUNCTIONAL ANALYSIS:-

Normed spaces, Banach spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension, Bounded and continuous linear operators, Linear operators and functionals on finite dimensional spaces, Normed spaces of operators, Dual spaces, Hilbert spaces, Orthogonal complements and direct sums, Bessel's inequality, Total orthonormal sets and sequences, Representation of functionals on Hilbert spaces, Hilbert adjoint operators, Self-adjoint, unitary and normal operators. Hahn Banach theorems for real and complex normed spaces, Adjoint operator, Reflexive spaces, Uniform boundedness theorem strong and weak convergence, Convergence of sequences of operators and functionals, Open mapping theorem,

Closed graph theorem, Spectrum of an operator, Spectral properties of bounded linear operators, Nonemptiness of the spectrum, Spectral Properties of Bounded Linear Operators, Resolvent and Spectrum, Banach Algebras and its classifications for commutative case.

18. GROUP ACTIONS, COMPOSITION SERIES and FIELD THEORY:-

Definition and examples of group actions, Permutation representations; Centralizers and Normalizers, Stabilizers and kernels of group actions; Groups acting on themselves by left multiplication and conjugation with consequences; Cayley's theorem, Conjugacy classes, Class equation, Conjugacy in S_n, Simplicity of A_5. p-groups, Sylow p-subgroups, Sylow's theorem, Applications of Sylow's theorem, Groups of order pq and p^2q (i and q both prime); Finite simple groups, Nonsimplicity tests, Solvable groups and their properties, Commutator subgroups, Nilpotent groups, Composition series, Jordan-Hölder theorem.

Fields and their extensions, Splitting fields, Normal extensions, Algebraic closure of a field, Separability, Perfect fields, Automorphisms of field extensions, Artin's theorem, Galois extensions, Fundamental theorem of Galois theory, Roots of unity, Cyclotomic polynomials and extensions, Finite fields, Theorem of primitive element and Steinitz's theorem.

19. NUMERICAL ANALYSIS AND MATHEMATICAL PROGRAMMING

Methods for Solving Algebraic and Transcendental Equations, Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method and Secant method, Linear Systems, Partial and scaled partial pivoting, LU decomposition and its applications, Iterative methods: Gauss−Jacobi, Gauss−Seidel and SOR methods, Lagrange and Newton interpolation, Piecewise linear interpolation.

Numerical Differentiation and Integration, First and higher order approximation for first derivative, Approximation for second derivative, Richardson extrapolation method; Numerical integration by closed Newton Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Second order Runge−Kutta Methods: Modified Euler's method, Heun's method and optimal RK2 method.

Mathematical programming problems with nonlinear objective and nonlinear constraints: Existence theorems, first order optimality conditions and second order optimality conditions for unconstrained optimization problems, Convex functions, Differentiable convex functions, Optimization on convex sets, Separation theorems, Fritz John optimality conditions for constrained nonlinear programming problems, Constraint qualifications, Karush-Kuhn Tucker conditions in nonlinear programming, Second order conditions in nonlinear programming, Langragian saddle points, Duality in nonlinear programming, Strong duality in convex programming, duality for linear and quadratic problems.

20. INTEGRAL TRANSFORMS:-

Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of

Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

21. CALCULUS OF SEVERAL VARIABLES:-

Basic concepts, Limits and continuity, Partial derivatives, Tangent planes, Total differential, Differentiability, Chain rules, Directional derivatives and the gradient, Extrema of functions of two variables, Method of Lagrange multipliers with one constraint

Double integration over rectangular and nonrectangular regions, Double integrals in polar coordinates, Triple integrals over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals, vector field, Divergence and curl, Line integrals and applications to mass and work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Functions of Several Variables: Linear Transformation, The Space of Linear Transformations on R^n to R^m as a Metric Space. Differentiation of Vector-valued Functions, Differentiation of a Vector-valued Function of Several Variables, Partial Derivatives, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem.

22. PARTIAL DIFFERENTIAL EQUATIONS:-

Solution of first order PDE, Classification of Second Order Linear PDE, Gravitational potential, Conservation laws and Burger's equations, Classification of second order PDE, Reduction to canonical forms, Equations with constant coefficients, General solution, Cauchy Problem and Wave Equations, Mathematical modeling of vibrating string and vibrating membrane, Cauchy problem for second order PDE, Homogeneous wave equation, Initial boundary value problems, Nonhomogeneous boundary conditions, Finite strings with fixed ends, Non-homogeneous wave equation, Goursat problem, Method of separation of variables for second order PDE, Vibrating string problem, Existence, and uniqueness of solution of vibrating string problem, Heat conduction problem, Existence and uniqueness of solution of heat conduction problem, Non-homogeneous problem.

Fourier transform and its application to solution of PDEs, Boundary value problems, Maximum and minimum principles, Uniqueness and continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method, Solution of Dirichlet problem for a circle in form of Poisson integral formula, Theory of Green's function for Laplace equation in two dimension and application in solution of Dirichlet and Neumann problem for half plane and circle, Theory of Green's function for Laplace equation in three dimension and application in solution of Dirichlet and Neumann problem for semi-infinite spaces and spheres.

Wave equation, Helmholtz's first and second theorems, Green's function for wave equation, Duhamel's principles for wave equation, Diffusion equation, Solution of initial boundary value problems for diffusion equation, Green's function for diffusion equation, Duhamel's principles for heat equation.

23. COMPUTATIONAL METHODS FOR ODEs AND PDEs:-

Initial Value Problems (IVPs) for the system of ODEs, Difference equations, Numerical Methods, Local truncation errors, Global truncation error, Stability analysis, Interval of absolute stability, Convergence and consistency, Single-step methods, Taylor series method, Explicit and implicit Runge Kutta methods and their stability and convergence analysis, Extrapolation method, Runge Kutta method for the second order ODEs and Stiff system of differential equations. Multi-step methods, Explicit and implicit multi-step methods, General linear multi-step methods and their stability.

Finite difference methods for 2D and 3D elliptic boundary value problems (BVPs) of second approximations, Finite difference approximations to Poisson's equation in cylindrical and spherical polar coordinates, Solution of large system of algebraic equations corresponding to discrete problems and iterative methods (Jacobi, Gauss-Seidel and SOR), Alternating direction methods.
Explicit and implicit finite difference approximations to heat conduction equation with Dirichlet and Neumann boundary conditions, Stability analysis, compatibility, consistency and convergence of the difference methods, Finite difference approximations to heat equation in polar coordinates.

24. TOPOLOGY:-

Topological spaces, Basis and subbasis for a topology, Order topology, Subspaces, Continuous functions, Homeomorphism, Product topology, Connected spaces, Components, Path connected spaces, Local connectedness, Local path-connectedness, Convergence, Sequences and nets, Hausdorff spaces, 1st and 2nd countable spaces, Separable and Lindelöf spaces, Compactness, Tychonoff's theorem, Quotient spaces and examples, Identification maps, cones, suspensions, local compactness and one-point compactification, Proper maps, Regularity, Complete regularity, Stone-Cech compactification, Normality, Urysohn lemma, Tietze extension theorem, Urysohn metrization theorem, Paracompactness, Partition of unity.

25. INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS:-

Definitions of Integral Equations and their classification, Eigen values and Eigen functions. Reduction to a system of algebraic equations, An Approximate Method. Fredholm Integral equations of the first kind, Method of Successive Approximations, Iterative Scheme for Volterra and Fredholm Integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution, Resolvent kernel and its results. Application of iterative Scheme to Volterra integral equations of the Second kind, Classical Fredholm Theory: Method of solution of Fredholm equations, Fredholm Theorems, Symmetric Kernels, Introduction to Complex Hilbert Space, Orthonormal system of functions, Riesz-Fischer Theorem, Fundamental properties of Eigen values and Eigen functions for symmetric kernels, Expansion in Eigen function and bilinear form, Hilbert Schmidt Theorem and some immediate consequences, Solutions of integral equations with symmetric kernels, Singular Integral Equations: The Abel integral equation, Cauchy principal value for integrals, Cauchy type integrals, singular integral equation with logarithmic kernel, Hilbert- kernel, solution of Hilbert-type singular integral equation, Calculus of Variations, Variational problems, the variation of a functional and its properties, Extremum of a functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a function and variable end point problem.

26. CRYPTOGRAPHY

Cryptography and Data Encryption Standard, Computer security concepts, Security attacks, Symmetric cipher model, Cryptanalysis and brute-force attack, Substitution techniques, Caesar

cipher, Monoalphabetic ciphers, Playfair cipher, Hill cipher, Polyalphabetic ciphers, One-time pad, Transposition techniques, Binary and ASCII, Pseudo-random bit generation, Stream ciphersand Block ciphers, Feistal cipher, Data encryption standard (DES), DES example, Algorithms and Advanced Encryption Standard, Finite Fields, divisibility, polynomial and modular arithmetic, Fermat's and Euler's theorems, Chinese remainder theorem, Discrete logarithm, Finite fields of the form GF(p) and GF($2n$); Advanced encryption standard (AES),AES transformation functions, AES key expansion, AES example, Principles of public-key cryptosystems, RSA algorithm and security of RSA, Elliptic curve arithmetic, Elliptic curve cryptography, Cryptographic Hash functions, Secure Hash algorithm, Digital Signatures and network Security, Digital signatures, Elgamal and Schnorr digital signature schemes, Digital signature algorithm. Wireless network and mobile device security, Email architecture, formats, threats and security, Secure/Multipurpose Internet Mail Extension, Pretty Good Privacy.