COMMON SYLLABUS FOR OBJECTIVE TYPE SUBJECT APTITUDE TEST (SAT) FOR RECRUITMENT TO POSTS OF LECTURER (SCHOOL NEW) IN MATHEMATICS, CLASS-III IN THE DEPARTMENT OF HIGHER EDUCATION. THIS PAPER WILL BE OF 02 HOURS DURATION OF 100 MARKS. THE OBJECTIVE TYPE SUBJECT APTITUDE TEST (SAT) SHALL COVER FOLLOWING TOPICS.

## PART-I (subject) 80 marks

## 1. REAL ANALYSIS:

Sequences and Series:Convergent Sequences, Sub-sequences, Cauchy Sequences (in metric spaces), Absolute Convergence, Addition and Multiplication of Series, Rearrangements of Series of Real Numbers, Power series, Uniqueness Theorem for Power Series. Limits and Continuity: Continuity, Limits of Functions (in Metric Spaces), Continuous Functions, Uniform Continuity, Compactness. Differentiation: The Derivative of a real function, Mean value theorems, The Continuity of Derivatives, L'Hôpital's Rule, Derivatives of Higher Order, Limit Inferior and Limit Superior, Integral Test, Comparison Test. Riemann-Stieltjes Integral: Definition and Existence of RiemannStieltjes Integral, Properties of the Integral, Integration and Differentiation, The Fundamental Theorem of Calculus, Change of Variable. Rectifiable Curves, Sequences and Series of Functions, Problem of Interchange of Limit Processes for Sequences of Functions, Pointwise and Uniform Convergence, Cauchy Criterion for Uniform Convergence, Weierstrass M-Test, Abel's and Dirichlet's Tests for Uniform Convergence, Uniform Convergence and Continuity, Uniform Convergence and Riemann-Stieltjes Integration, Uniform Convergence and Differentiation, The Weierstrass Approximation Theorem.

## 2. Algebra:

Permutations, Combinations, Pigeonhole Principle, Inclusion-Exclusion Principle, Derangements, Fundamental Theorem of Arithmetic, Divisibility in Z, Congruences, ChineseRemainder Theorem, Euler's $\phi$-function, Primitive Roots, Groups, Subgroups, Normal Subgroups, Quotient Groups, Homomorphisms, Cyclic Groups, Permutation Groups, Conjugacy and G-Sets, Normal Series, Solvable Groups, Nilpotent Groups, Direct Products, Finitely Generated Abelian Groups, Invariants of a Finite Abelian Groups, Cayley's Theorem, Class Equations, Sylow Theorems. Rings, Ideals, Prime and Maximal Ideals, Quotient Rings, Unique Factorization Domain, Principal Ideal Domain, Euclidean Domain, Polynomial Rings and Irreducibility Criteria. Fields, Finite Fields, Field Extensions, Galois Theory.

## 3. COMPLEX ANALYSIS:

Complex Numbers, Polar Representation, Stereographic Projections, Square and Square Root Functions, Exponential Function, Logarithm Function, Power Functions and Phase Factors, Trigonometric and Hyperbolic Functions. Analytic Functions, Cauchy-Riemann Equations, Inverse Mappings and the JacobianHarmonic Functions, Conformal Mappings, Fractional Linear Transformations, Line Integrals and Harmonic Functions, Line Integrals and Green's Theorem. Independence of Path, Harmonic Conjugates, Mean Value Property, Maximum Modulus Principle.Complex Line Integrals, Fundamental Theorem of Calculus for Analytic Functions, Cauchy's Theorem, Cauchy Integral Formula, Liouville's Theorem, Morera's Theorem, Goursat's Theorem, Power Series, Power Series Expansion of an Analytic Function, Manipulation of Power Series, Zeros of an Analytic Function, Taylor and Laurent Series. Isolated Singularities and Residue Calculus,Laurent Decomposition, Isolated Singularities of an Analytic Function, Isolated Singularity at Infinity, Partial Fractions Decomposition, Residue Theorem, Integrals Featuring Rational Functions, Integrals of Trigonometric Functions.

## 4. Ordinary Differential Equations (ODEs):

Systems of Linear Differential Equations: System of First-Order Equations, Existence and Uniqueness Theorem, Fundamental Matrix, Non-homogeneous Linear Systems, Linear Systems with Constant Coefficients, Linear Systems with Periodic Coefficients. Existence and Uniqueness of Solutions: Successive Approximations, Picard's Theorem, Continuation and Dependence on Initial Conditions, Existence of Solutions in the Large Interval, Existence and Uniqueness of Solutions of Systems, Fixed Point Method. Boundary Value Problems:Sturm-Liouville Problem, Green's Function. Applications of Boundary Value Problems, Picard's Theorem. Oscillations of Second-Order Equations:Sturm's Comparison Theorem and Separation Theorem, Elementary Linear OscillationsComparison Theorem. Stability of Linear and Nonlinear Systems: Elementary Critical Points, System of Equations with Constant Coefficients, Linear Equation with Constant Coefficient, Lyapunov Stability, Stability of Quasi-Linear Systems, Second-Order Linear Differential Equations.

## 5. Partial Differential Equations (PDEs):

Partial Differential Equations of the First Order: Origins of First Order Partial Differential Equations, Cauchy's Problem for First Order Equations, Linear Equations of the First Order, Nonlinear Partial Differential Equations of the First Order, Cauchy's Method of Characteristics, Charpit's Method, Jacobi's Method. Partial Differential Equations of the Second Order: Linear Partial Differential Equations with Constant Coefficients, Characteristic Curves of Second Order Equations, Characteristics of Equations in Three Variables, Solution of Linear Hyperbolic Equations, Separation of Variables. Laplace's Equation: Occurrence of Laplace's Equation in Physics, Elementary Solution of Laplace's Equation, Boundary Value Problems, Separation of Variables in the Two-Dimensional Laplace Equation. The Wave Equation:, Occurrence of the Wave Equation in Physics, Elementary Solutions of the One-Dimensional Wave Equation, Green's Function for the Wave Equation.

## 6. LINEAR ALGEBRA:

General Vector Spaces: Real Vector Spaces, Subspaces, Linear Independence, Basis and Dimension, Row Space, Column Space, and Null Space, Rank and Nullity. Inner Product Spaces: Inner Products, Angle and Orthogonality in Inner Product Spaces, Orthonormal Bases, GramSchmidt Process, QR-Decomposition, Best Approximation, Least Squares, Change of Basis, Orthogonal Matrices. Linear Transformations: General Linear Transformations, Kernel and Range, Inverse Linear Transformations, Matrices of General Linear Transformations, Similarity, Isomorphism. Eigenvalues and Eigenvectors:Eigenvalues and Eigenvectors, Orthogonal Matrices, Unitary Matrices, Normal Matrices and Hermitian Matrices, Similar Matrices, Properties of Eigenvalues and Eigenvectors, Diagonalization, Factorization. Application to Differential Equations, Approximation Problems, Fourier Series and Quadratic Forms.

## 7. Topology:

Topological Spaces:Open Bases and Open Subbases, Weak Topologies, Functional Algebras C(X,R) and C(X,C). Compactness:Compact Spaces, Product of Spaces, Tychonoff's Theorem and Locally Compact Spaces, Compactness for Metric Spaces, Ascoli's Theorem. Separation Axioms:T1-Spaces and Hausdorff Spaces, Completely Regular Spaces and Normal Spaces, Urysohn's Lemma and the Tietze Extension Theorem, The Urysohn Imbedding Theorem, The Stone-Cech Compactification. Connectedness: Connected Spaces, Components of a Space, Totally Disconnected Spaces, Locally Connected Spaces.

## 8. Functional Analysis:

Banach Spaces:Definition and Some Examples, Continuous Linear Transformations, The HahnBanach Theorem, The Open Mapping Theorem, The Closed Graph Theorem, The Uniform Boundedness Principle, The Natural Embedding of N in $\mathrm{N}^{* *}$, Reflexivity. Hilbert Spaces:Definition and Some Simple Properties, OrthogonalComplements, Orthonormal Sets, The Conjugate Space H*. The Adjoint of an Operator, Self-Adjoint, Normal, and Unitary Operators, Projections. Spectral Theory of Linear Operators in Normed Spaces:Spectral Theory in FiniteDimensional Normed Spaces, Basic Concepts, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent and Spectrum, Use of Complex Analysis in Spectral Theory, Banach Algebras, Further Properties of Banach Algebras.

## 9. ANALYTICAL NUMBER THEORY:

Divisibility Theory in the Integers:The Division Algorithm, The Greatest Common Divisor, The Euclidean Algorithm, The Diophantine Equation ax+by =c. Primes and Their Distribution: The Fundamental Theorem of Arithmetic, The Sieve of Eratosthenes, The Goldbach Conjecture. The Theory of Congruences: Basic Properties of Congruence, Special Divisibility Tests and Linear Congruences. Fermat's Theorem:Fermat's Factorization Method, The Little Theorem, Wilson's Theorem. Number-Theoretic Functions: The Functions $\tau$ and $\sigma$. The Möbius Inversion Formula, The Greatest Integer Function and an Application to the Calendar. Euler's Generalization of Fermat's Theorem:Euler's Phi-Function, Euler's Theorem and Some Properties of the Phi-Function, An Application to Cryptography. Primitive Roots and Indices: The Order of an Integer Modulo n, Primitive Roots for Primes, Composite Numbers Having Primitive Roots, The Theory of Indices. The Quadratic Reciprocity Law:Euler's Criterion, The Legendre Symbol and Its Properties, Quadratic Reciprocity and Quadratic Congruences with Composite Moduli.

## 10. OPERATIONS RESEARCH:

Operations Research (OR):History of OR, Scientific Methods, Modeling in OR, OR Models, Methodology of OR, OR in Decision Making, Applications of OR. Convex Sets and Their Properties:Convex Sets, Hyperplane and Hyperspheres, Open and Close Half-Spaces, Theorem on Convex Sets, Convex Polyhedron, Feasible, Basic Feasible, and Optimal Solutions, Extreme Points. Linear Programming Problem (LPP):Mathematical Formulation of LPP, Graphical Solution of LPP, Simplex Method, Charnes Big M Method, Two-Phase Method, Degeneracy, Unrestricted Variables, Unbounded Solutions, Revised Simplex Method (Standard Form-I). Duality Theory:Concept of Duality in LPP, Dual LPP, Fundamental Properties of Dual Problems, Duality Theorems, Complementary Slackness, Dual Simplex Algorithm, Advantages of Duality. Integer Programming (IPP):Pure and Mixed IPP, Gomory's Method, Geometrical Interpretation of Cutting Plane Method, Branch and Bound Method. Transportation Problem (TP):Mathematical Formulation, Basic Feasible Solutions of TPs by North-West Corner Method, Least Cost Method, Vogel's Approximation Method, Unbalanced TP, Optimality Test of Basic Feasible Solution (BFS) by U-V Method, Degeneracy in TP. Assignment Problem (AP):, Mathematical Formulation, Assignment Methods, Hungarian Method, Unbalanced AP, Rule to Draw Minimum Numbers of Lines, Illustrative Problems, Traveling Salesman Problem. Game Theory:Two-Person, Zero-Sum Games, The Maximin-Minimax Principle, Pure Strategies, Mixed Strategies, Graphical Solution of 2xn and $m \times 2$ Games, Dominance Property, General Solution of $m \times n$, Rectangular Games, Linear Programming Problem of GP. Queueing Theory:Queueing Systems, Queueing Problem, Transient and Steady States, Probability Distributions in Queueing Systems, Poisson Process (Pure Birth Process), Properties of Poisson's Arrivals, Exponential Process, Markovian Property, Pure Death Process, Service Time Distribution, Erlang service time distribution, Solution of Queueing Models: ( $\mathrm{M} \mid(\mathrm{M} \mid 1):(\infty \mid \mathrm{FCFS})$, (Birth and Death Model).

## 11. INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS:

Integral Equations: Definitions of Integral Equations and their classification, Eigen values and Eigen functions, Reduction to a system of algebraic equations, An Approximate Method, Fredholm Integral equations of the first kind. Method of Successive Approximations: Iterative Scheme for Volterra and Fredholm Integral equations of the second kind, Conditions of uniform convergence and uniqueness of series solution, Resolvent kernel and its results. Application of iterative Scheme to Voltaire integral equations of the Second kind. Classical Fredholm Theory: Method of solution of Fredholm equations, Fredholm Theorems. Symmetric Kernels: Introduction to Complex Hilbert Space, Orthonormal system of functions, Riesz-Fischer Theorem, Fundamental properties of Eigen values and Eigen functions for symmetric kernels, Expansion in Eigen function and bilinear form, Hilbert Schmidt Theorem and some immediate consequences, Solutions of integral equations with symmetric kernels. Calculus of Variations: Variational problems, the variation of a functional and its properties, Extremum of a functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a function and variable end point problem.

## 12. Probability and Statistics :

Probability: Sample Spaces and Events, Basic Set Theory, Definitions of Probability, Axioms of Probability, Addition Theorem, Conditional Probability, Multiplicative Theorem, Total Probability, Bayes ‘Theorem, Independent Events. Random Variables:Introduction, Types of Random Variables, Discrete Random Variables, Continuous Random Variables, Probability Distribution Function, Properties of Distribution Function, Probability Density Function (PDF), Properties of Density Function, Mathematical Expectation, Variance, Chebyshev's Inequality. Probability and Distribution:Binomial Distribution, Poisson Distribution, Uniform Distribution, Normal Distribution. Regression and Correlation: Correlation Analysis, Types of Correlation, Positive and Negative Correlation, Simple, Partial, and Multiple Correlation, Linear and Non-Linear Correlation, Methods of Studying Correlation, Scatter Diagram Correlation, Graphic Method, Karl Pearson's Coefficient of Correlation, Rank Correlation, Partial Correlation, Multiple Correlation, Regression Analysis. Tests of Statistical Hypothesis - Large Sample Tests: Introduction, Statistical Hypothesis, Test of a Statistical Hypothesis, Procedure for Testing of Hypothesis, Set up a Hypothesis, Set up a Suitable Significance Level, Setting Test Criterion, Doing Computations, Making Decisions, Type I and Type II Errors, Two-Tailed and One-Tailed Test of Hypothesis, Test of Significance of Single Mean, Test of Significance for Difference of Standard Deviation, Test of Significance for Single Proportion, Test of Significance for Difference of Proportions. Tests of Statistical Hypothesis-Small Sample Tests:Introduction, Student's t-Distribution, Properties of t-Distribution, Test of Significance of Single Mean, Test of Significance for Inference Between Two Mean of Independent Samples, Test of Significance for Difference Between Two Means (Dependent Samples), Test of Significance of an Observed Correlation Coefficient, F-Test for Equality of Population Variance. Chi-Square Distribution: Properties of Chi-Square Distribution, Application of Test of a Statistical Hypothesis, Chi-Square Test for Goodness of Fit, Conditions for Applying Chi-Square Test, Degree of Freedom, Chi-Square Test for Independence of Attributes. Time Series Analysis: Significance of Time Series Analysis, Components of Time Series. Secular Trend: Freehand or Graphic Method, Semi-Average Method, Method of Moving Averages, Method of Least Squares, Straight Linear and Non-Linear Trends. Seasonal Variations: Method of Simple Averages, Ratio-to-Trends Method, Ratio-to-Moving Average Method, Link Relative Method.

## NUMERICAL ANALYSIS:

Transcendental and Polynomial Equations: Bisection Method, Iteration Methods based on First Degree Equation, Rate of Convergence, General Iteration Methods, Methods for Complex Roots, Polynomial Equations, Choice of an Iterative Method and Implementation, Problems. System of Linear Algebraic Equations and Eigenvalue Problems:Introduction, Direct Methods, Error Analysis for Direct Methods, Iteration Methods, Eigenvalues and Eigenvectors, Bounds on Eigenvalues, Jacobi Method for Symmetric Matrices, Givens Method for Symmetric Matrices, Householder's Method for Symmetric Matrices, Power Method, Inverse Power Method, Choice of a Method. Interpolation:Introduction, Lagrange and Newton Interpolations, Finite Difference Operators, Interpolating Polynomials using Finite Differences, Hermit Interpolation, Piecewise and Spline Interpolation, Least Squares Approximation. Differentiation and Integration:Introduction, Numerical Differentiation, Optimum Choice of Step Length, Extrapolation Methods, Partial Differentiation, Numerical Integration, Methods Based on Interpolation, Methods Based on Undetermined Coefficients, Composite Integration Methods, Romberg Integration, Double Integration, Problems. Ordinary Differential Equations: Introduction, Numerical Methods, SingleStep Methods, Multi-Step Methods, Predictor-Corrector Methods, Stability Analysis of Multi-Step Methods, Stiff Systems, Boundary Value Problems, Initial Value Problems.

## 13. MEASURE THEORY AND INTEGRATION:

Functions of Several Variables: Linear Transformation, the Space of Linear Transformations on $\mathrm{R}^{\mathrm{n}}$ to $\mathrm{R}^{\mathrm{m}}$ as a Metric Space. Differentiation of Vector-valued Functions, Differentiation of a Vectorvalued Function of Several Variables, Partial Derivatives, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem, Differentiation and Integration Introduction on Differentiation of Monotone Functions, Functions of Bounded Variation. Differentiation of an Integral, Absolute Continuity, Convex Functions. Lebesgue Measure:

Introduction, Outer measure, Measurable sets and Lebesgue measure, Countable Additivity, Anonmeasurable set, Measurable functions, Littlewood's three principles. The Lebesgue Integral: The Riemann integral, The Lebesgue Integral of Simple Functions, The Lebesgue Integral of a Bounded Function Over a Set of Finite Measure, The Integral of Nonnegative Functions, The General Lebesgue Integral, Convergence in Measure.

## OR

## Cryptography:

Mathematics of Cryptography: Integer Arithmetic, Modular Arithmetic, Matrices, Linear Congruence. Traditional Symmetric-Key Ciphers: Substitution Ciphers, Transposition Ciphers, Stream and Block Ciphers. Mathematics of Symmetric-Key Cryptography: Algebraic Structures, $\operatorname{GF}\left(2^{\mathrm{n}}\right)$ Fields. Introduction to Modern Symmetric-Key Ciphers: Modern Block Ciphers, Modern Stream Ciphers. Data Encryption Standard (DES): DES Structure, DES Analysis, Security of DES, Multiple DES-Conventional Encryption Algorithm. Advanced Encryption Standard (AES): Transformations, Key Expansion, The AESCiphers, Analysis of AES. Encipherment Using Modern Symmetric-KeyCiphers: Use of Modern Block Ciphers, Use of Stream Ciphers. Mathematics of Asymmetric-Key Cryptography: Primes, Primality Testing, Factorization, Chinese Remainder

Theorem, Quadratic Congruence, Exponentiation and Logarithm. Asymmetric-Key Cryptography: RSA Cryptosystem, Rabin Cryptosystem, ElGamal Cryptosystem, Elliptic Curve Cryptosystem.

## 14. CLASSICAL MECHANICS:

Frame of References: Different Coordinate Systems (Cartesian, Plane Polar, Cylindrical, and Spherical Polar Coordinate System), Generalized Coordinates, Constraints, Work and Potential Energy, Generalized Forces, The Principle of Virtual Work. Introduction to Lagrange's Equations:Lagrange's Equations for a Particle in a Plane. The Classification of Dynamical Systems, Lagrange's Equations for Any Simple Dynamical System, Lagrange's Equations for NonHolonomic Systems with Moving Constraints, Lagrange's Equations for Impulsive Motion. The Variational Principle/Hamilton's Principle: Lagrange's Equation from Hamilton's Principle, Stationary Values of a Function, Constrained Stationary Values, Stationary Value of a Definite Integral, The Brachistochrone Problem, Hamilton's Equations, Derivation of Hamilton's Equations, Ignorable Coordinates, The Routhian Function, The Form of Hamiltonian Function, Modified Hamilton's Principle, Principle of Least Action (Different Form of Least Action Principle), The Hamilton-Jacobi Equation, Lagrange and Poisson Brackets, Calculus of Variation, Invariance of Lagrange and Poisson Brackets under Canonical Transformation.

## OR

## DIFFERENTIAL GEOMETRY:

Curves: Tangent, Principal Normal, Curvature, Binormal, Torsion, Serret-Frenet Formulae, Locus of Center of Curvature, Spherical Curvature, Locus of Center of Spherical Curvature, Curve Determined by Its Intrinsic Equations, Helices, Involutes \& Evolutes. Surfaces: Tangent Plane, Normal, Curvilinear Co-ordinates, First-Order Magnitudes, Directions on a Surface, The Normal, Second-Order Magnitudes, Derivatives of n, Curvature of Normal Section, Meunier's Theorem, Principal Directions and Curvatures, First and Second Curvatures, Euler's Theorem. Special Surfaces: Surface of Revolution Gauss's Formulae for $\rho_{11,} \rho_{12,} \rho_{22}$, Gauss Characteristic Equation, Mainardi-Codazzi Relations, Derivatives of Angle $\omega$, Geodesic Property, Equations of Geodesics, Torsion of Geodesic, Bonnet's Theorem, Vector Curvature, Geodesic Curvature $\kappa_{g}$.

## Part-II (B.Ed. portion) 20 marks

## Foundation of Education System

- Concept and Nature of Knowledge, Knowledge Acquiring Process.
- Indian Knowledge System; Vedic Education System, Buddhist Education System
- Concept of Education; Meaning, Nature, Modes of Education, Objectives of Education, Approaches to Education.
- Philosophical, Psychological, Sociological and Technological Basis of Education ( Meaning and Needs)
- Concept of important Variables: Personality, Intelligence, Creativity, Academic Achievement, Problem Solving Ability


## Teacher Education and Related Aspects:-

- Concept of Teacher Education, Historical Aspects of Teacher Education in India, Scope of Teacher Education, Types and Levels of Teacher Education Programmes.
- Teacher and Teaching related Concepts, Learner and Learning Related Concepts, Relation between Teaching and Learning, Learning Environment, Pillars of Learning.
- Challenges in the field of Teacher Education (Discussion with reference to NEP 2020 also)


## Pedagogy and Teaching Learning Experiences:-

- Communication Skills and its Use
- Models of Teaching; Advance organizer, Concept Attainment, Information Processing, Inquiry Training
- Methods of Teaching
- Preparation and Use of Teaching Learning Material


## Information Technology in Teaching Learning Process

- ICT; Its meaning and use in Teaching Learning Process
- Concept of Open Educational Resources, SWAYAM and MOOCs
- Digital Initiatives in School Education by Government of India

