TEST BOOKLET

AP(AS&H) PHYSICS-3/2014

Time Allowed : 2 Hours

[Maximum Marks : 100

All questions carry equal marks.

INSTRUCTIONS

1. Immediately after the commencement of the examination, you should check that test booklet does not have any unprinted or torn or missing pages or items, etc. If so, get it replaced by a complete test booklet.

2. Write your Roll Number only in the box provided alongside. Do not write anything else on the Test Booklet.

3. This Test Booklet contains 100 items (questions). Each item comprises four responses (answers). Choose only one response for each item which you consider the best.

4. After the candidate has read each item in the Test Booklet and decided which of the given responses is correct or the best, he has to mark the circle containing the letter of the selected response by blackening it completely with Black or Blue ball pen. In the following example, response “C” is so marked:

5. Do the encoding carefully as given in the illustrations. While encoding your particulars or marking the answers on answer sheet, you should blacken the circle corresponding to the choice in full and no part of the circle should be left unfilled.

6. You have to mark all your responses ONLY on the ANSWER SHEET separately given according to INSTRUCTIONS FOR CANDIDATES’ already supplied to you. Responses marked on the Test Booklet or in any paper other than the answer sheet shall not be examined.

7. All items carry equal marks. Attempt all items. Your total marks will depend only on the number of correct responses marked by you in the Answer Sheet. There will be no negative marking.

8. Before you proceed to mark responses in the Answer Sheet fill in the particulars in the front portion of the Answer Sheet as per the instructions sent to you.

9. After you have completed the test, hand over the Answer Sheet to the Invigilator.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

P.T.O.
1. Time-dependent Schrodinger equation for a free particle in one-dimension is:

(A) \( i\hbar \frac{\partial}{\partial t} \varphi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \varphi(x, t) \)

(B) \( i\hbar \frac{\partial^2}{\partial t^2} \varphi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \varphi(x, t) \)

(C) \( i \frac{\partial^2}{\partial t^2} \varphi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial}{\partial x} \varphi(x, t) \)

(D) \( -\frac{\hbar^2}{2m} \frac{\partial}{\partial t} \varphi(x, t) = i\hbar \frac{\partial}{\partial x} \varphi(x, t) \)

2. The probability current density, \( \vec{j}(r, t) \) is given by:

(A) \( \frac{i\hbar}{2m} (\varphi \nabla \varphi^* - \varphi^* \nabla \varphi) \)

(B) \( i (\varphi \nabla \varphi^* - \varphi^* \nabla \varphi) \)

(C) \( i\hbar^2 (\varphi \nabla \varphi^* + \varphi^* \nabla \varphi) \)

(D) \( i\hbar m (\varphi \nabla \varphi^* + \varphi^* \nabla \varphi) \)
3. A particle constrained to move along $x$-axis in the domain $0 \leq x \leq L$ has a wave function $\psi(x) = \sin \left( \frac{nx}{L} \right)$, where $n$ is an integer. Find the normalization constant.

(A) $\sqrt{2}$

(B) $\sqrt{\frac{1}{L}}$

(C) $\sqrt{\frac{L}{\sqrt{2}}}$

(D) $\sqrt{\frac{2}{L}}$

4. The mathematical representation of a spherical wave travelling outwards from a point is given by $\psi(r) = \frac{A}{r} e^{ikr}$, where $A$ is a constant and $k$ is the wave vector. Find the expression for the probability current density:

(A) $\frac{\hbar k}{mk r^2} |A|^2$

(B) $\frac{\hbar k}{m r^2} |A|^2$

(C) $\frac{\hbar^2 k}{kr^2} |A|^2$

(D) $\frac{\hbar k}{mr} |A|^2$

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5. The phase velocity of a relativistic particle of rest mass $m_0$, having de Broglie wavelength $\lambda$ is:

(A) $c \left( \frac{1 + m_0^2 c^2 \lambda^2}{\hbar^2} \right)^{1/2}$

(B) $c \left( \frac{1 - m_0^2 c^2 \lambda^2}{\hbar^2} \right)^2$

(C) $c \left( \frac{1 - m_0^2 c^2 \lambda^2}{\hbar^2} \right)^{1/2}$

(D) $c \left( \frac{1 + m_0^2 c^2 \lambda^2}{\hbar^2} \right)^2$

6. Electrons of energy 12.09 eV can cause radiation to be emitted from a hydrogen atom. Find the principal quantum number of the orbit to which electron in the hydrogen atom is excited:

(A) 5

(B) 4

(C) 3

(D) 2
7. The density and atomic weight of hydrogen are 0.08898 kg/m³ and 1.00797, respectively. What is the density (in kg/m³) of nitrogen of atomic weight 14.0067 ?

(A) 2.426  (B) 1.236  
(C) 1.007  (D) 0.976

8. What is the magnetic moment (in amp-m²) associated with the first orbit in the case of hydrogen atom?

(A) $2.31 \times 10^{-32}$  (B) $3.23 \times 10^{-33}$  
(C) $9.27 \times 10^{-34}$  (D) $0.27 \times 10^{-34}$

9. A photon of wavelength 140 nm is absorbed by mercury vapour and two other photons are emitted. If one of them is of 185 nm line, what is the wavelength of the other photon?

(A) 375.3 nm  (B) 413.7 nm  
(C) 437.9 nm  (D) 575.2 nm
10. Assuming an overlap interaction between nearest neighbours of the type
\[ \varphi(r) = B \exp\left(-\frac{r}{\rho}\right), \]
where \( B \) and \( \rho \) are constants, find the equilibrium spacing \( r_0 \) in terms of \( B \) and \( \rho \).

(A) \( \frac{\rho}{3} \log_{\epsilon} B \)  
(B) \( \rho \log_{\epsilon} B \)

(C) \( \frac{2}{3} \rho \log_{\epsilon} B \)  
(D) \( \frac{3}{2} \rho \log_{\epsilon} B \)

11. How many photons of radiation of wavelength \( 6 \times 10^{-12} \) metre must fall per second on a blackened plate to produce a force of one dyne?

(A) \( 7.6 \times 10^{21} \)  
(B) \( 3.6 \times 10^{23} \)

(C) \( 4.9 \times 10^{25} \)  
(D) \( 5.6 \times 10^{27} \)

12. An X-ray tube passes 10 mA current at a potential difference of 150 kV. Find the rate at which the target is being heated if only 1% of the incident energy is converted into X-rays.

(A) 2100 W  
(B) 1923 W

(C) 1785 W  
(D) 1485 W
13. The mass absorption coefficient of iron for X-rays of wavelength 0.013 nm is 0.04 m²/kg. What thickness of iron is required to reduce the X-ray intensity to 1/1000 of its original value? Take, density of iron = 8000 kg/m³.

(A) 0.02 m  (B) 0.05 m

(C) 0.07 m  (D) 0.09 m

14. Find the de Broglie wavelength of a 10 eV electron.

(A) 0.543 nm  (B) 0.488 nm

(C) 0.388 nm  (D) 0.198 nm

15. A beam of neutrons with energies ranging from zero to several electron volts is directed at a crystal with a grating space of 0.303 nm. Find the angle between the incident beam and the crystal so that the reflected neutrons will have a kinetic energy of 0.1 eV.

(A) \(\sin^{-1} (0.23)\)  (B) \(\sin^{-1} (0.15)\)

(C) \(\cos^{-1} (0.09)\)  (D) \(\cos^{-1} (0.25)\)
16. If a proton is confined to a nucleus of radius $5 \times 10^{-15}$ metre, what is the value of its minimum kinetic energy?

(A) 0.21 MeV

(B) 0.32 MeV

(C) 0.45 MeV

(D) 0.17 MeV

17. An electron is confined to move between two rigid walls separated by 1.0 nm. Find the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies?

(A) 1.2 nm, 0.83 nm, 0.45 nm

(B) 5.6 nm, 3.6 nm, 2.3 nm

(C) 2.0 nm, 1.0 nm, 0.67 nm

(D) 0.68 nm, 0.53 nm, 0.33 nm
18. How many atoms per mm$^2$ surface area are there in (1, 1, 0) plane for lead which has fcc structure. The radius of the atoms is 0.174 nm.

(A) $1.3 \times 10^{-19}$  \hspace{1cm} (B) $3.3 \times 10^{17}$

(C) $7.3 \times 10^{14}$  \hspace{1cm} (D) $5.8 \times 10^{12}$

19. For a simple cubic lattice, compare the density of points in (1, 1, 1) and (1, 1, 0) planes:

(A) $\sqrt{1} : \sqrt{3}$  \hspace{1cm} (B) $\sqrt{2} : \sqrt{3}$

(C) $1 : 3$  \hspace{1cm} (D) $2 : 3$

20. What is the lowest energy of an electron confined in a box of each side 0.1 nm?

(A) 112.9 eV  \hspace{1cm} (B) 167.3 eV

(C) 198.2 eV  \hspace{1cm} (D) 213.4 eV
21. A particle of mass 1.0 $\mu$g is confined to move between two right walls separated by a distance of 0.1 mm. If the particle requires 100s to cross the gap. What quantum number describes this motion?

(A) $10^{12}$  
(B) $2 \times 10^{13}$  
(C) $3 \times 10^{14}$  
(D) $4 \times 10^{15}$

22. The Fermi energy of silver is 5.51 eV. What is the average energy of the free electrons in silver at 0 K?

(A) 6.93 eV  
(B) 5.03 eV  
(C) 4.13 eV  
(D) 3.31 eV

23. At what frequency will there be a 0.01\% change in the conductivity of a material from the static value if the relaxation time is of the order of $10^{14}$ sec.

(A) 160 GHz  
(B) 279 GHz  
(C) 382 GHz  
(D) 523 GHz
24. Find the critical current which can flow through a long thin superconducting wire of aluminium of diameter $10^{-3}$ meter. The critical magnetic field for aluminium is $7.9 \times 10^3$ amp/metre.

(A) 26.5 A  
(B) 24.8 A  
(C) 21.4 A  
(D) 18.9 A  

25. The critical temperature for some metal with isotopic mass 100 is 4.2 K. Find its critical temperature when its isotopic mass changes to 127:

(A) 3.80 K  
(B) 5.26 K  
(C) 6.78 K  
(D) 7.99 K  

26. A magnetic material has a magnetization of 3300 amp/metre and flux density of 0.0044 weber/meter$^2$. Find the magnetising force of the material:

(A) 413 A/m  
(B) 375 A/m  
(C) 309 A/m  
(D) 203 A/m.
27. The variation of \( \ln \sigma \) of a semiconductor with \( T^{-1} \) is shown in the figure (symbols have their usual meanings). Find the band gap.

\[
\begin{array}{c}
\hline
\text{ln} \sigma \\
\hline
\text{7} \\
\text{6} \\
\text{5} \\
\text{4} \\
\text{3} \\
\hline
1/T \\
\hline
\end{array}
\]

\[1.2 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0 \times 10^{-3}\]

(A) 0.39 eV  
(B) 0.43 eV  
(C) 0.58 eV  
(D) 0.36 eV

28. An electric field of 100 V/m is applied to a sample of n-type semiconductor whose Hall coefficient is \(-0.0125 \text{ m}^3/\text{coulomb}\). Find the current density in the sample, assuming \( \mu_n = 0.36 \text{ m}^2\text{V}^{-1}\text{s}^{-1} \).

(A) 2880 A/m²  
(B) 2020 A/m²  
(C) 1639 A/m²  
(D) 1023 A/m²
29. A spaceship moving away from earth with velocity of 0.5c fires a rocket whose velocity relative to spaceship is 0.8c away from the earth. What will be the velocity of the rocket as observed from the earth?

(A) 0.31 c  (B) 0.41 c  
(C) 0.51 c  (D) 0.71 c

30. Find the minimum wavelength of radiation for which diamond can be transparent, given the energy gap for diamond to be 6.0 eV.

(A) 109 nm  (B) 207 nm  
(C) 413 nm  (D) 512 nm

31. A laser beam of wavelength 7400 Å has coherence time $4 \times 10^{-5}$ sec. Find the order of magnitude of spectral half width:

(A) 0.45 Å  (B) 0.55 Å  
(C) 0.61 Å  (D) 0.72 Å
32. A particle of mass \( m \) moves under the influence of the potential 
\[ V(x) = \frac{A}{x^2} - \frac{B}{x}, \] where \( A, B > 0 \). Find the frequency of small oscillations around the equilibrium point:

(A) \[ \sqrt{\frac{B^2}{8mA^2}} \]

(B) \[ \sqrt{\frac{B^4}{8mA^3}} \]

(C) \[ \sqrt{\frac{8B^4}{3mA^3}} \]

(D) \[ \sqrt{\frac{A^3}{3mB^4}} \]

33. A mass \( m \) with speed \( v \) approaches a stationary mass \( M \). The masses bounce off each other without any loss in total energy. What are the final velocities of the particles?

(A) \[ v_m = \frac{m - M}{m + M} v; \quad v_M = \frac{2mv}{m + M} \]

(B) \[ v_m = \frac{m + M}{m - M} v; \quad v_M = \frac{2mv}{m - M} \]

(C) \[ v_m = \frac{2mv}{m + M} v; \quad v_M = \frac{m - M}{m + M} \]

(D) \[ v_m = \frac{m - M}{m + M} v; \quad v_M = \frac{2Mv}{m + M} \]
34. Sand drops vertically from a negligible height at a rate $\sigma$ kg/s onto a moving conveyor belt. If the conveyor belt is forced to move at a constant speed $v$, how much energy is lost to heat per unit time?

(A) $\frac{\sigma v^2}{2}$  (B) $\frac{\sigma v^2}{3}$

(C) $\sigma v^2$  (D) $\frac{\sigma v^2}{6}$

35. The Lagrangian of a particle of mass $m$ sliding off a fixed frictionless hemisphere of radius $R$ (shown in figure) is:

![Diagram of a hemisphere with a particle sliding off it.](image)

(A) $L = mR \dot{\theta}^2 - mgR \sin \theta$

(B) $L = \frac{1}{2} mR \dot{\theta}^2 - mgR \sin \theta$

(C) $L = \frac{1}{2} mR^2 \dot{\theta}^2 - mgR \cos \theta$

(D) $L = \frac{1}{2} mR \dot{\theta}^2 - \frac{1}{2} mgR \cos \theta$
36. Consider a mass \( m \) on a spring, with relaxed length zero, in the \( x-y \) plane.

The Lagrangian \( L = \frac{1}{2} m (x^2 + y^2) - \left( \frac{k}{2} \right) (x^2 + y^2) \), is invariant under the change of coordinates, \( x \rightarrow x + \epsilon y \) and \( y \rightarrow y - \epsilon x \), to first order in \( \epsilon \). Find the conserved momentum.

(A) \( m(x\dot{y} - y\dot{x}) \)  
(B) \( m(\dot{x}y - \dot{y}x) \)  
(C) \( m(\dot{x} + \dot{y}) \)  
(D) \( m(3\dot{x} + 3\dot{y}) \)

37. The inertia tensor for a solid cube of mass \( M \) and side length \( L \), with the coordinate axes parallel to the edges of the cube and the origin at a corner, is:

\[
\begin{pmatrix}
\frac{2}{3} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{3}{4}
\end{pmatrix}
\]

(A) \( ML^2 \)  
(B) \( ML^2 \)  
(C) \( \frac{1}{2} ML^2 \)  
(D) \( \frac{1}{2} ML^2 \)
38. Two trains, A and B each have proper length \( L \) and move in the same direction. A's speed is \( 4c/5 \) and B's speed is \( 3c/5 \). A starts behind B. How long, as viewed by an observer on the ground, does it take for A to overtake B?

(A) \( \frac{L}{c} \)  
(B) \( \frac{3L}{c} \)  
(C) \( \frac{5L}{c} \)  
(D) \( \frac{7L}{c} \)

39. A particle of mass \( M \) and energy \( E \) decays into two identical particles A and B. In the lab frame, A and B are emitted, as shown. What are the energies of the created particles?

\[ (A) \quad E_A = \frac{M^2}{2E} \quad E_B = \frac{2E^2 - M^2}{2E} \]

\[ (B) \quad E_A = \frac{2E^2 - M^2}{2E} \quad E_B = \frac{M^2}{2E} \]

\[ (C) \quad E_A = M \quad E_B = E - M \]

\[ (D) \quad E_A = \frac{M^2}{E} \quad E_B = \frac{E^2 - M^2}{E} \]
40. A particle of mass $m$ acted upon by a restoring force $F = -mk/x^3$ towards the centre of the force O, where $x$ is the distance of the particle from O and $k$ the force constant. If the particle begins its motion from a point at a distance $d$ from O, how much time it will take to reach at O?

(A) $\frac{d^4}{k}$  \hspace{1cm} (B) $\frac{d^2}{\sqrt{k}}$

(C) $\frac{d}{k^{1/4}}$  \hspace{1cm} (D) $\frac{d}{k}$

41. The electromagnetic force experienced by a particle having charge $q$ and moving with velocity $\vec{v}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$ is:

(A) $\vec{F} = q(\vec{E} - \vec{v} \times \vec{B})$

(B) $\vec{F} = q(\vec{E} - \vec{B} \times \vec{v})$

(C) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

(D) $\vec{F} = q(\vec{B} - \vec{v} \times \vec{E})$
42. For the potential energy \( U = \frac{1}{r} + \frac{r^2}{rc^2} \), the expression for the corresponding force is:

(A) \( F = \frac{1}{r^2} \left( 1 - \frac{r^2 - 2rr'}{c^2} \right) \)

(B) \( F = \frac{1}{r^2} \left( 1 - \frac{r^2 - rr'}{c^2} \right) \)

(C) \( F = \frac{1}{r^2} \left( 1 + \frac{r^2 - 2rr'}{c^2} \right) \)

(D) \( F = \frac{1}{r^2} \left( 1 + \frac{r^2 - 2rr'}{c^2} \right) \)

43. A planet is revolving around the sun in an elliptical orbit with semi-major axis 'a' and semi-minor axis 'b'. The ratio of the maximum and the minimum speeds of the planet is:

(A) \( \frac{a + b}{a - b} \)

(B) \( \frac{\sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2}} \)

(C) \( \frac{a - \sqrt{a^2 + b^2}}{a + \sqrt{a^2 - b^2}} \)

(D) \( \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \)
44. A particle of mass $m$ is thrown vertically upward with a velocity $u$ from the earth's surface. If the frictional force of the air is $mkv^2$ (where $v$ is the instantaneous velocity and $k$ the force constant), the time taken by the particle to reach the highest point is:

(A) $\frac{1}{\sqrt{gk}} \tan^{-1} \left( \frac{u\sqrt{k}}{\sqrt{g}} \right)$

(B) $\frac{1}{\sqrt{gk}} \sin^{-1} \left( \frac{u\sqrt{k}}{\sqrt{g}} \right)$

(C) $\frac{1}{gk} \tan^{-1} \left( \frac{u\sqrt{k}}{\sqrt{g}} \right)$

(D) $\frac{1}{gk} \sin^{-1} \left( \frac{u\sqrt{k}}{\sqrt{g}} \right)$

45. Two particles P and Q of masses $m_1$ and $m_2$ respectively, move under their gravitational attraction (as shown). The Lagrangian equations for the motion of these particles are:

\[ \ddot{x}_1 = \frac{Gm_2}{(x_2 - x_1)^2}; \quad \ddot{x}_2 = \frac{Gm_1}{(x_2 - x_1)^2} \]

(A) \[ \ddot{x}_1 = \frac{Gm_2}{(x_2 - x_1)^2}; \quad \ddot{x}_2 = \frac{Gm_1}{(x_2 - x_1)^2} \]

(B) \[ \ddot{x}_1 = \frac{-Gm_2}{(x_2 - x_1)^2}; \quad \ddot{x}_2 = \frac{-Gm_1}{(x_2 - x_1)^2} \]

(C) \[ \ddot{x}_1 = \frac{-Gm_1}{(x_2 - x_1)^2}; \quad \ddot{x}_2 = \frac{-Gm_2}{(x_2 - x_1)^2} \]

(D) \[ \ddot{x}_1 = \frac{Gm_1}{(x_2 - x_1)^2}; \quad \ddot{x}_2 = \frac{-Gm_2}{(x_2 - x_1)^2} \]
46. The Lagrangian of a particle is given by \( L = \frac{1}{2} m \left( \dot{r}^2 + r^2 \dot{\theta}^2 \right) - V(r) \). Find the generalized momenta.

(A) \( p_r = \frac{1}{2} m \dot{r}; p_\theta = \frac{1}{2} m r^2 \dot{\theta} \)

(B) \( p_r = \frac{3}{2} m \dot{r}; p_\theta = \frac{1}{2} m r^2 \dot{\theta} \)

(C) \( p_r = m \dot{r}; p_\theta = m r^2 \dot{\theta} \)

(D) \( p_r = \sqrt{2} m \dot{r}; p_\theta = \sqrt{2} m r^2 \dot{\theta} \)

47. Which of the following properties is satisfied by Poisson brackets?

(A) \( [f, g]_{q,p} = -[g, f]_{q,p} = [g, f]_{p,q} \)

(B) \( [f, c]_{q,p} \neq 0, c \) is a constant

(C) \( [f_1 + f_2, g]_{q,p} = [f_1, g]_{q,p} = [g, f_2]_{p,q} \)

(D) \( \frac{\partial}{\partial t} [f, g]_{q,p} = \left[ \frac{\partial f}{\partial t}, g \right]_{q,p} + \left[ f, \frac{\partial g}{\partial t} \right]_{q,p} \)
48. If a particle describes an orbit \( r = \alpha e^{\beta t} \), the force on it varies as:

(A) \( \frac{1}{r} \)  
(B) \( \frac{1}{r^2} \)  
(C) \( \frac{1}{r^3} \)  
(D) \( \frac{1}{r^4} \)

49. For a particle moving under the inverse square law of force, with kinetic energy \( T \) and potential energy \( V \), which of the following relations is satisfied by \( T \) and \( V \)?

(A) \( 2T + V = 0 \)  
(B) \( T + V = \text{constant} \)  
(C) \( T + V = 0 \)  
(D) \( T - V = 0 \)

50. The half-life time of a radioactive particle measured in the laboratory is \( 4.0 \times 10^{-8} \)s when its speed is \( 0.8c \). Find the half-life time when its speed is \( 0.6c \).

(A) \( 8.0 \times 10^{-8} \)s  
(B) \( 4.0 \times 10^{-8} \)s  
(C) \( 3.0 \times 10^{-8} \)s  
(D) \( 2.2 \times 10^{-8} \)s

51. At what speed a clock should be moved so that it may appear to lose 1 minute in each hour?

(A) \( 5.5 \times 10^7 \)m/s  
(B) \( 5.6 \times 10^6 \)m/s  
(C) \( 3.9 \times 10^5 \)m/s  
(D) \( 2.6 \times 10^4 \)m/s
52. Find the velocity of an electron so that its momentum is 10 times that of the product of its rest mass and the speed of light (c):

(A) $0.563c$  
(B) $0.695c$

(C) $0.837c$  
(D) $0.995c$

53. A point $P$ is represented in the $(x_1, y_1, z_1)$ system by $P(1, 2, 3)$. In another coordinate system, the same point is represented as $P(x'_1, y'_1, z'_1)$ where $y_1$ has been rotated towards $z_1$ around the $x_1$-axis by an angle of $30^\circ$. Find the rotation matrix:

(A) \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix}
\]

(B) \[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\
0 & \frac{1}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix}
\]

(C) \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix}
\]

(D) \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & \frac{\sqrt{3}}{2}
\end{pmatrix}
\]

54. Two position vectors are represented in cartesian coordinates as $\vec{A} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 3\hat{k}$. Find the angle between $\vec{A}$ and $\vec{B}$.

(A) $90^\circ$  
(B) $60^\circ$

(C) $45^\circ$  
(D) $30^\circ$
55. Consider the one-dimensional potential, \( U(x) = \frac{-wd^2(x^2 + d^2)}{x^4 + 8d^4} \). Find the turning points for \( E = -\frac{\omega}{8} \). The value of \( \omega \) is positive.

(A) \( \pm 2\sqrt{2}d, 0 \)  
(B) \( \pm 2\sqrt{2}d, 1 \)  
(C) \( \pm 2\sqrt{2}d, \pm 1 \)  
(D) \( 2\sqrt{2}d, 0 \)

56. A sawtooth driving force function is shown in the figure. Express \( F(t) \) as a Fourier series:

\[
\begin{align*}
F(t) & = \frac{A}{\pi} \left[ \sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t - \ldots \right] \\
F(t) & = A \left[ \sin \omega t + \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t + \ldots \right] \\
F(t) & = \frac{A}{\pi} \left[ \sin \omega t - \sin 2\omega t + \sin 3\omega t + \ldots \right] \\
F(t) & = A \left[ \sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t - \ldots \right]
\end{align*}
\]
57. Halley's comet, which passed around the sun in 1986, moves in a highly elliptical orbit with an eccentricity of 0.97 and a period of 76 years. Find its maximum distance from the sun. (Given mass of the sun $= 2.0 \times 10^{30}$ kg; $G = 6.7 \times 10^{-11}$ Nm$^2$/kg$^2$).

(A) $6.9 \times 10^{15}$ m  
(B) $6.2 \times 10^{13}$ m  
(C) $5.3 \times 10^{12}$ m  
(D) $2.9 \times 10^{11}$ m

58. For the common base circuit, as shown, find the values of $I_c$ and $V_{CB}$.

(Take: $V_{BE} = 0.7$ V)

(A) 4.9 mA; 12.2 V  
(B) 3.2 mA; 10.6 V  
(C) 2.5 mA; 8.8 V  
(D) 1.9 mA; 5.7 V
59. Group the following nuclei as isotones:

\[ ^{12}\text{C}, ^{13}\text{C}, ^{14}\text{N}, ^{14}\text{O}, ^{15}\text{N}, ^{15}\text{O}, ^{16}\text{O}, ^{16}\text{O}, ^{17}\text{N}, ^{17}\text{O} \]

(A) \[ ^{15}\text{N}, ^{15}\text{O} \]

(B) \[ ^{13}\text{C}, ^{14}\text{N}, ^{15}\text{O} \]

(C) \[ ^{17}\text{N}, ^{17}\text{O} \]

(D) \[ ^{14}\text{O}, ^{15}\text{O}, ^{16}\text{C} \]

60. A singly charged positive ion is accelerated through a potential difference of 1000 V in a mass spectrograph. It then passes through a uniform magnetic field \( B = 1500 \text{ Gauss} \), and then deflected into a circular path of radius \( 0.122 \text{ m} \). What is the mass number of the ion?

(A) 8

(B) 12

(C) 16

(D) 40
61. For the mirror nuclei $^{23}_{11}$Na and $^{23}_{12}$Mg, the approximate value of the Coulomb coefficient is:

$$[M(\text{Na}) = 22.989 \text{ amu}, M(\text{Mg}) = 22.994 \text{ amu}, M(n) = 1.008 \text{ amu}, M(p) = 1.007 \text{ amu}]$$

(A) 0.13 MeV  (B) 0.36 MeV  
(C) 0.49 MeV  (D) 0.66 MeV

62. On the basis of shell model, the magnetic moment (in the units of nuclear magneton) of $^{19}_{10}$Ne nucleus is:

(A) 4.79  (B) 3.76  
(C) 2.29  (D) 1.76

63. Which of the following reactors produced maximum neutron flux?

(A) Apsara  (B) Zerlina  
(C) Dhurva  (D) Kamini
64. The scattering of an energetic charged particle in matter is mostly due to interaction with:

(A) electrons  (B) pions

(C) nuclei  (D) quarks

65. An X-ray photon of frequency $3.2 \times 10^{19}$ Hz collides with an electron and gets scattered through $60^\circ$. What is the frequency (in Hz) of scattered photon?

(A) $9.92 \times 10^{19}$  (B) $2.47 \times 10^{19}$

(C) $1.02 \times 10^{18}$  (D) $6.28 \times 10^{17}$

66. Find the velocity of electrons having kinetic energy of 1.022 MeV using relativistic considerations:

(A) $0.94 \, c$  (B) $0.92 \, c$

(C) $0.89 \, c$  (D) $0.82 \, c$
67. In a 70 MeV betatron synchrotron, the radius of the stable orbit is 23 cm.

Find the value of magnetic field at this orbit for the given energy:

(A) 0.64 T       (B) 0.72 T

(C) 0.83 T       (D) 0.91 T

68. Arrange the following particles according to their masses, greatest first:

\[ p, \Lambda^0, \Sigma^0, \Xi^- \]

(A) \( \Xi^-, \Sigma^0, \Lambda^0, p \)       (B) \( p, \Lambda^0, \Sigma^0, \Xi^- \)

(C) \( \Lambda^0, p, \Xi^-, \Sigma^0 \)       (D) \( \Lambda^0, \Xi^-, \Sigma^0, p \)

69. What is the quark composition of \( \Sigma^- \)?

(A) uus       (B) uda

(C) dds       (D) uss
70. Which of the following is satisfied by the Pauli matrices \( \sigma_i \)?

(A) \( \sigma_i \sigma_j + \sigma_j \sigma_i = 2\sigma_k \)

(B) \( \sigma_i \sigma_j - \sigma_j \sigma_i = 2i\sigma_k \)

(C) \( \sigma_i \sigma_j - \sigma_j \sigma_i = 2\sigma_k \)

(D) \( \sigma_i \sigma_j + \sigma_j \sigma_i = 2i\sigma_k \)

71. Image of \(|z + 1|\) under the mapping \( w = \frac{1}{z} \) is:

(A) \( 2v + 1 = 1 \)

(B) \( 2v - 1 = 0 \)

(C) \( 2u + 1 = 0 \)

(D) \( 2u - 1 = 0 \)

72. Using the method of residues, find the value of \( \oint_C \frac{1}{\sinh z} \, dz \) where \( C \) is the circle \(|z| = 4 : \)

(A) \(-2\pi i\)  \hspace{10cm}  (B) \( \pi i \)

(C) \(-\pi i\)  \hspace{10cm}  (D) zero
73. A sample of certain element is placed in a magnetic field of 0.6 T and suitably excited. How far apart are the Zeeman components of the 450 nm spectral line of this element?

(A) 0.00566 nm
(B) 0.00632 nm
(C) 0.00715 nm
(D) 0.00856 nm

74. The operator $|\psi \rangle \sim |\phi \rangle$ is a projection operator only when:

(A) $|\phi \rangle$ is not normalized
(B) $|\phi \rangle$ is purely imaginary
(C) $|\phi \rangle$ is real
(D) $|\phi \rangle$ is normalized

75. If $a'$ and $|\phi \rangle$ are, respectively, the eigenvalue and eigenfunction of an operator $O$, the eigenvalue of an operator $e^{ia}$ for the same eigenfunction would be:

(A) $ia$
(B) $a$
(C) $e^{ia}$
(D) $e^{a}$
76. A generator develops 200 V and has an internal resistance of 100 Ω. Find the efficiency of the generator if the power is delivered to a load of 300 Ω.

(A) 25%  
(B) 50%  
(C) 75%  
(D) 100%

77. An a.c. supply of 230 V is applied to a half-wave rectifier circuit through a transformer of turn ratio 10 : 1. Find the output d.c. voltage.

(A) 5.49 V  
(B) 7.13 V  
(C) 10.36 V  
(D) 11.52 V

78. A 7.2 V zener is used in the circuit, as shown, and the load current is to vary from 12 to 100 mA. Find the value of series resistance R to maintain a voltage of 7.2 V across the load. The input voltage is constant at 12 V and the minimum zener current is 10 mA.

(A) 43.5 Ω  
(B) 38.1 Ω  
(C) 34.3 Ω  
(D) 23.8 Ω
79. When negative voltage feedback is applied to an amplifier of gain 100, the overall gain falls to 50. What is the fraction of the output voltage feedback?

(A) 2  (B) 0.5
(C) 0.01  (D) 0.02

80. The r.m.s. value of carrier voltage is 100 V. After amplitude modulation by a sinusoidal a.f. voltage, the r.m.s. value becomes 110 V. Find the modulation index.

(A) 0.65  (B) 0.53
(C) 0.49  (D) 0.36

81. Who was the last ruler of Kangra princely state?

(A) Ghamand Chand  (B) Sansar Chand
(C) Anirudh Chand  (D) Dhruv Dev Chand
82. In which month is Renuka Fair celebrated?

(A) Kartika
(B) Baishakh
(C) Shrawan
(D) Bhadon

83. Near which place in Bilaspur District of H.P. is Markandeya Shrine and holy spring?

(A) Jhandutta
(B) Jeori Pattan
(C) Jukhala
(D) Jagatkhana

84. In which year was the Judicial Commissioner's Court in H.P. replaced by the Delhi High Court?

(A) 1963
(B) 1967
(C) 1971
(D) 1972

85. According to 2011 Census which district of H.P. has the lowest sex ratio?

(A) Solan
(B) Kinnaur
(C) Bilaspur
(D) Chamba
86. Which region of H.P. is Halda Festival celebrated?

- (A) Chandra-Bhaga Valley
- (B) Doon Valley
- (C) Kunihar Valley
- (D) Balh Valley

87. Which one of the following is a domed style temple?

- (A) Jawalamukhi (Kangra District)
- (B) Mrikula Devi (Udaipur, Lahaul-Spiti District)
- (C) Tripura Sundri (Naggar, Kullu District)
- (D) Lakshna Devi (Bharmaur, Chamba District)

88. Who traced the origin of Pahari language to Dardi and Pishachi?

- (A) Bhola Nath Tiwari
- (B) Hardev Bahri
- (C) G.A. Grierson
- (D) Govind Chatak
89. At which educational level is scholarship given to girl students belonging to Balmiki families under Maharshi Balmiki Chhatravriti Yojna?

(A) Primary level

(B) Elementary level

(C) Upto matric level

(D) Beyond matric level upto college level

90. With the collaboration of which country was Dhauladhar Farm Forestry Project started in 1980?

(A) Australia  
(B) France

(C) Germany  
(D) USA

91. Whom did the Indian men’s Kabaddi team beat in the final of the 17th Asian Games held at Incheon, South Korea?

(A) Afghanistan  
(B) Japan

(C) Iran  
(D) Malaysia
95. Under the old rules of International Hockey Federation there used to be 15 minutes break after half time. What is the new rule?

(A) Five minutes break after every 15 minutes of play

(B) Five minutes break after first and third quarter of play and 15 minutes break after half time

(C) Five minutes break after first and third quarter of play and 10 minutes break at half time

(D) Ten minutes break after each quarter

96. At what age did Malala Yousafzai win the 2014 Nobel Peace Prize?

(A) 15

(B) 17

(C) 20

(D) 21

97. Who is the author of *The Dreams of My Father*?

(A) Varun Gandhi

(B) Vini Mahajan

(C) Barack Obama

(D) Bilawal Bhutto
98. Which country is at number one in the global slavery Index of 2014 ?

(A) China       (B) India

(C) Pakistan    (D) Nigeria

99. To which country did aid worker Kassig who was helping Syrian refugees in Turkey and whom the Jihadis claim to have executed around November, 2014, belong ?

(A) France      (B) Britain

(C) Russia      (D) USA

100. In which year was Right Livelihood Award which is treated as alternative Nobel Prize instituted ?

(A) 1980        (B) 1996

(C) 2004        (D) 2013