HIMACHAL PRADESH
PUBLIC SERVICE COMMISSION

SCREENING TEST FOR THE POST OF LECTURER APPLIED SCIENCES AND HUMANITIES (POLYTECHNIC) MATHEMATICS (CLASS-I GAZETTED) IN THE DEPARTMENT OF TECHNICAL EDUCATION, H.P.

TIME ALLOWED: 2.00 HOURS. MAXIMUM MARKS: 100

Write your Roll. No.

Note: All questions carry equal marks. Out of four options given at the end of each question, please indicate the correct option.

1. Let \( A = \{1, 2, 3, \ldots, 100\} \). Then \( A \) has
(a) 100 accumulation points
(b) at least one accumulation point
(c) no accumulation point
(d) the number 100 as the only accumulation point.

2. The function \( f(x, y) = \sqrt{|xy|} \) is
(a) not differentiable at \((0,0)\) but the partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) exist at the origin
(b) differentiable at \((0,0)\) and the partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) exist at the origin
(c) differentiable at \((0,0)\) but the partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) do not exist at the origin
(d) not differentiable at \((0,0)\) and also the partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) do not exist at the origin

3. A set \( A \) in a topological space \( X \) is said to be compact if
(a) every open cover of \( A \) has a countable subcover
(b) every open cover of \( A \) has a finite subcover
(c) there exists an open cover of \( A \) which has a finite subcover
(d) every open cover of \( A \) has a finite subcover.

4. Bolzano-Weierstrass theorem states that in \( \mathbb{R}^n \), every
(a) unbounded infinite set has a limit point
(b) bounded infinite set has no limit point
(c) bounded infinite set has a limit point
(d) bounded finite set has a limit point.

5. Which of the following is NOT correct
(a) Arbitrary union of open sets is open
(b) Arbitrary intersection of open sets is open
(c) Finite intersection of closed sets is closed
(d) Finite union of closed sets is closed
6. The function \( f(x) = \tan x \) is:
(a) analytic in \( S \)
(b) analytic in \( |x| < \pi \)
(c) analytic in \( |x| > \pi \)
(d) analytic except for poles

7. Let \( A \) be the open interval \((3,4)\) and let \( B \) be the closed interval \([5,6]\) in the complex plane \( \mathbb{C} \). Then
(a) \( A \) is open set and \( B \) is closed set
(b) \( A \) is closed set and \( B \) is also a closed set
(c) \( A \) is open set and \( B \) is also an open set
(d) None of the above

8. Let \( S = A \cup \{2\} \) where \( A \) is the interval \((-1,1)\) in the real number system \( \mathbb{R} \). Then 2 is:
(a) adherent point of \( S \) and also isolated point of \( S \)
(b) adherent point of \( S \) but not isolated point of \( S \)
(c) isolated point of \( S \) but not adherent point of \( S \)
(d) neither isolated point of \( S \) nor adherent point of \( S \)

9. The function \( f(x) = \frac{\sin x}{x} \) has
(a) removable singularity at 0
(b) pole at 0
(c) non-isolated singularity at 0
(d) essential singularity at 0.

10. The function \( f(x) = \csc x \) has
(a) residue \( R(f, 0) = 2 \pi \)
(b) residue \( R(f, 0) = 2 \pi i \)
(c) residue \( R(f, 0) = 1 \)
(d) \( R(f, 0) = 0 \)

11. The radius of convergence of the power series \( \sum_{n=0}^{\infty} \frac{z^{4n}}{1+4n} \) is
(a) 0
(b) 1
(c) 4
(d) \( \infty \)

12. Let \( y \) be the closed contour given by \( y(t) = \frac{5 \pi}{2} e^{it}, 0 \leq t \leq 2 \pi \). Then \( \int_{y} \cot z \, dz \) is
(a) 10
(b) 10 \( \pi \)
(c) 10 \( \pi i \)
(d) \( \infty \)
13. The series $\sum_{k=1}^{\infty} \frac{k}{e}$
(a) converges and also converges absolutely
(b) converges but does not converge absolutely
(c) does not converge and also does not converge absolutely
(d) converges absolutely but does not converge.

14. A set $E$ is said to be Lebesgue measurable, if for each set $A$ and outer measure $m^*$,
(a) $m^*(A) = m^*(A \cap E) + m^*(A \cap E^c)$
(b) $m^*(A) = m^*(A \cap E) \cup m^*(A \cap E^c)$
(c) $m^*(E) = m^*(A \cap E) + m^*(A \cap E^c)$
(d) $m^*(E) = m^*(A \cap E) \cup m^*(A \cap E^c)$.

15. Let $f(x) = \frac{|x|}{x}$ for $x \neq 0$ and $f(0) = 0$. Then
(a) $f$ is continuous at 0
(b) $f$ has removable discontinuity at 0
(c) $f$ has jump discontinuity
(d) $f$ has discontinuity of the second kind.

16. The limit superior and limit inferior respectively of the sequence $\left\{ \sin \frac{n\pi}{2} \right\}_{n \in \mathbb{N}}$ is
(a) 0, 1
(b) 1, −1
(c) 1, 0
(d) 0, −1

17. Let $A = \{ x \in \mathbb{C} : |z| < 2 \} \cup \{ x \in \mathbb{C} : |z| > 3 \}$. Then
(a) $A$ is closed set
(b) $A$ is open set
(c) $A$ is closed set as well as open set
(d) $A$ is neither open set nor closed set

18. In a discrete metric space $(X, d)$,
(a) $d(x, x) > 0$
(b) $d(x, y) = 0$ if $x \neq y$
(c) $d(x, y) = 0$ if $x = y$
(d) $d(x, y) = 1$ if $x = y$

19. Every $T_1$ topological space is
(a) $T_1$ space
(b) $T_2$ space and also regular
(c) $T_2$ space but not regular
(d) None of the above
20. Let \( C \) be a circle of with center \( \Pi \) and radius \( \varepsilon \). Then \( \int_{\Gamma} \frac{1}{z-\Pi} \, dz = \) 
(a) 0 
(b) \( \varepsilon \) 
(c) \( -\Pi \) 
(d) None of the above.

21. Let \( f(z) = \frac{z^{3}}{z-1} \). Then Residue of \( f \) at \( z = 2 \) is 
(a) 0 
(b) -4 
(c) 4 
(d) None of the above.

22. The value of \( \int_{\gamma} \sin x \, dx \), where \( \gamma \) is the circle \( x = 2 \), is 
(a) 0 
(b) \( \pi \) 
(c) \( 2 \pi \) 
(d) None of the above.

23. The permutation \( (1, 2, 3, 4, 5, 6, 7, 8, 9) \) of the set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) can be written as the product of disjoint cycles and product of transpositions respectively as 
(a) \( (1, 2, 3)(4, 5)(6, 7, 8, 9) \) 
(b) \( (1, 2, 3)(4, 5)(6, 7, 8, 9) \) 
(c) \( (1, 2, 3)(4, 5)(6, 7, 8, 9) \) 
(d) \( (1, 2, 3)(4, 5)(6, 7, 8, 9) \).

24. The number of generators of cyclic group of order 12 are 
(a) 1 
(b) 2 
(c) 3 
(d) 4.

25. Let \( G = \{ e, a, b, c \} \) and \( \langle c \rangle = G \). Then \( \langle a \rangle \) is a Klein's 4-group. Then \( e = a \) and \( 0 = \Phi \) respectively equal 
(a) e 
(b) b 
(c) c 
(d) \( c \).

26. Which of the following is NOT true: 
(a) Every Euclidean domain is a Principal ideal domain. 
(b) Every Principal ideal domain is a Euclidean domain. 
(c) Every Euclidean domain is a unique factorization domain. 
(d) For any field \( F \), the polynomial ring \( F[x] \) is a Euclidean domain.
27. If the Euler's $\phi$-function satisfies $\phi(n) = s$, then $s$ is 
(a) number of positive integers prime to $n$  
(b) number of positive integers relatively prime to $n$ 
(c) number of positive integers less than equal to $n$ which are relatively prime to $n$. 
(d) number of positive integers which divide $n$. 

28. Which of the following is NOT true.
(a) Every field is an integral domain 
(b) Every finite field is an integral domain 
(c) Every integral domain is a field 
(d) Every finite integral domain is a field. 

29. The integral surface of the PDE $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 1, y_0(s) = 0$ and $z_0(s) = s$ is 
(a) $x^2 + y^2 - xz - y + z = 1$ 
(b) $x^2 + yz - zx - y + z = 1$ 
(c) $x^2 + y^2 - xz + yz = 1$ 
(d) $x^2 + xz - xy + yz + z = 1$ 

30. For the initial value problem $y' = f(x, y), y(0) = 0$, $x \in [0,1]$ with $f(x, y) = \sqrt{y} + 1$, which of the following statements is true? 
(a) $f$ satisfies Lipchitz condition near origin 
(b) $\frac{\partial f}{\partial y}$ is bounded near origin 
(c) The above IVP has a unique solution. 
(d) The above IVP has more than one solution. 

31. The integral equation $y(x) = 1 + \lambda \int_{0}^{\frac{x}{\pi}} \cos(x - t)y(t)dt$ has 
(a) A unique solution for $\lambda \neq \frac{4}{\pi + 2}$ 
(b) A unique solution for $\lambda \neq \frac{4}{\pi - 2}$ 
(c) Infinitely many solutions for $\lambda = \frac{4}{\pi + 2}$ 
(d) No solution for $\lambda = \frac{4}{\pi + 2}$
32. The solution of the integral equation \( y(x) = x + \int_0^x (t-x)y(t) \, dt \) is
(a) \( \cos x - \sin x \)
(b) \( \cos x + \sin x \)
(c) \( \sin x \)
(d) \( \cos x \)

33. Let \( S_1 = 1 \), and \( S_{n+1} = \sqrt{3} S_n \), \( n = 1, 2, \ldots \). Then the sequence \( \{ S_n \} \) converges to
(a) 0
(b) 3
(c) \( \sqrt{3} \)
(d) 9

34. The function \( f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & \text{if } x^2 + y^2 \neq 0 \\ 0 & \text{if } x = y = 0 \end{cases} \)
is
(a) not continuous, possesses partial derivative, and is not differentiable at the origin
(b) continuous, does not possesses partial derivative, but is differentiable at the origin
(c) continuous, does not possesses partial derivative, and is not differentiable at the origin
(d) continuous, possesses partial derivative, but is not differentiable at the origin.

35. Let \( V = \mathbb{R}^3 \). Which of the following are linearly independent
(a) \( (0, 0, 0), (1, 1, 1), (2, 2, 2) \)
(b) \( (1, 1, 0), (1, 1, 0), (1, 1, 0) \)
(c) \( (2, 0, 0), (0, 2, 0), (0, 0, 2) \)
(d) \( (0, 0, 0), (0, 1, 0), (0, 0, 1) \)

36. In an inner product space, the Cauchy Schwarz inequality states that
(a) \( |x + y| \leq ||x|| + ||y|| \)
(b) \( ||x + y|| \leq ||x|| + ||y|| \)
(c) \( <x, y>| \leq ||x|| \cdot ||y|| \)
(d) \( |<x, y>| \leq ||x|| + ||y|| \)

37. Let \( L \) be a linear operator of a vector space \( V \) into itself. If \( L(v) = \lambda v \) and \( v \neq 0 \), then
(a) \( \lambda \) is called eigen value
(b) \( \lambda \) is called eigen vector
(c) \( \lambda v \) is called eigen value
(d) \( L \) is called eigen value.

38. The rank of the matrix \[
\begin{bmatrix}
0 & 2 & 3 & 1 \\
1 & 4 & 6 & 3 \\
3 & 3 & 7 & 5
\end{bmatrix}
\]is
(a) 1
(b) 2
(c) 3
(d) 4
39. Let $W_1$ and $W_2$ be finitely generated subspaces of a vector space $V$. Then
(a) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$
(b) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 + \dim(W_1 \cap W_2)$
(c) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$
(d) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cup W_2)$

40. The Taylor series expansion of $f(z) = \frac{z-1}{z+1}$ about $z = 0$ is
(a) $2(1 - z + z^2 - z^3 + \cdots)$
(b) $-1 + 2(z - z^2 + z^3 - \cdots)$
(c) $1 - 2(z - z^2 + z^3 - \cdots)$
(d) $1 + 2(z - z^2 + z^3 - \cdots)$

41. For any complex number $z$, $\sin(iz) =$
(a) $\frac{e^{iz} - e^{-iz}}{2i}$
(b) $\frac{e^{iz} - e^{-iz}}{2}$
(c) $\frac{e^{iz} - e^{-iz}}{z}$
(d) $\frac{e^{iz} - e^{-iz}}{2i}$

42. The bilinear transformation which maps the points 2, 1, $-\frac{1}{2}$ into the points 1, $i$, $-1$ is
(a) $\frac{3z - 2i}{i(z - 6)}$
(b) $\frac{3z - 2i}{iz - 6}$
(c) $\frac{3z + 2i}{2z - 6}$
(d) $\frac{3z + 2i}{iz + 6}$

43. Let $M$ be the set of all $2 \times 2$ matrices over integers under matrix multiplication. Then
(a) $M$ is a commutative ring without unity
(b) $M$ is a commutative ring with unity
(c) $M$ is a non commutative ring with unity
(d) $M$ is a non commutative ring with without unity
44. The mapping \( f(z) = e^z \) maps the complex plane \( \mathbb{C} \) onto.
(a) \(|z| < 1\)
(b) \(0 < |z| < 1\)
(c) \(\mathbb{C}\)
(d) None of the above

45. \( \log i = \)
(a) \(i \frac{\pi}{2}\)
(b) \(-i \frac{\pi}{2}\)
(c) \(\frac{\pi}{2}\)
(d) \(-\frac{\pi}{2}\)

46. Let \( G \) be a group and \( N \subseteq G \) (i.e., \( N \) be a normal subgroup of \( G \)). Let \( M \) be a subgroup of \( G \) such that \( N \subseteq N \) and \( M/N \subseteq G/N \). Then
(a) \( G/M \) is isomorphic to \( \frac{G/N}{M/N} \)
(b) \( G/N \) is isomorphic to \( \frac{G/M}{N/M} \)
(c) \( M/N \) is isomorphic to \( \frac{M/G}{M/N} \)
(d) \( M/N \) is isomorphic to \( \frac{M/G}{N/G} \)

47. Let \( G \) be a group of order 48. Then a 4-Sylow subgroup of \( G \) is of order
(a) 4
(b) 12
(c) 16
(d) 48

48. Solution of \( x^3 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 1 = 0 \) is
(a) Both bounded and periodic
(b) Periodic but not bounded
(c) Bounded but not periodic
(d) Neither bounded nor periodic.
19. Let \( u(x, y) \) be the solution of the Cauchy problem \( xu_x + u_y = 1, \quad u(x, 0) = 2 \log x, \ x > 1 \) then the value of \( u(e, 1) \) is

(a) 1
(b) \( e \)
(c) -1
(d) 0

50. The PDE: \( y \frac{\partial^2 u}{\partial x^2} + x^2 \frac{\partial^2 u}{\partial y^2} = 0 \) is elliptic in

(a) The first and third quadrants
(b) The second and fourth quadrants
(c) The first and second quadrants
(d) The third and fourth quadrants

51. The integral equation \( y(x) = 1 + \frac{1}{\pi} \int_0^{2\pi} \sin(x + t) y(t) dt \) has

(a) A unique solution
(b) Infinitely many solutions
(c) No solution
(d) Two solutions:

52. The functional \( \int_0^1 \left( y'^2 + 4y^2 + 8y e^x \right) dx, \quad y(0) = -\frac{4}{3}, y(1) = -\frac{4e}{3} \) possesses

(a) Strong minima on \( y = -\frac{4}{3} e^x \)
(b) Strong minima on \( y = -\frac{4}{3} e^x \)
(c) Weak maxima on \( y = -\frac{4}{3} e^x \)
(d) Strong maxima on \( y = -\frac{4}{3} e^x \)
53. Simpson's one-third rule for evaluation of \( \int_{a}^{b} f(x) \, dx \) requires the interval \([a, b]\) to be divided into:
(a) Any number of sub-intervals.
(b) Any number of sub-intervals of equal width.
(c) An even number of sub-intervals of equal width.
(d) An odd number of sub-intervals of equal width.

54. Let \( m, n \) be positive integers. Let \( V \) be a vector space spanned by \( m \) vectors. Then every \( n \) vector in \( V \) are linearly dependent if:
(a) \( n > m \)
(b) \( n < m \)
(c) \( n \geq m \)
(d) \( n \leq m \)

55. Let \( A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \), \( B = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix} \), \( C = \begin{bmatrix} -1 & 7 \\ 0 & 19 \end{bmatrix} \), \( u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), \( v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \), \( w = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \). Then:
(a) \( A, B, C \) are linearly independent and \( u, v, w \) are linearly dependent.
(b) \( A, B, C \) are linearly independent and \( u, v, w \) are linearly independent.
(c) \( A, B, C \) are linearly dependent and \( u, v, w \) are linearly dependent.
(d) \( A, B, C \) are linearly dependent and \( u, v, w \) are linearly independent.

56. If \( A = \begin{bmatrix} 5 & 2 \\ 1 & 7 \end{bmatrix} \), then the values for \( c_0, c_1, c_2 \) in the equation \( A^3 = c_0 I + c_1 A + c_2 A^2 \) respectively are:
(a) 72, -66, -17
(b) -72, 66, 17
(c) 72, 66, -17
(d) 72, -66, 17

57. The Hölder's inequality states that if \( \{x_n\}_{n=1}^{\infty} \) and \( \{y_n\}_{n=1}^{\infty} \) are sequences of real numbers and \( \frac{1}{p} + \frac{1}{q} = 1 \), then
(a) \( \sum_{n=1}^{\infty} |x_n y_n| \leq \left( \sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}} \left( \sum_{n=1}^{\infty} |y_n|^q \right)^{\frac{1}{q}} \)
(b) \( \sum_{n=1}^{\infty} |x_n + y_n| \leq \left( \sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}} + \left( \sum_{n=1}^{\infty} |y_n|^p \right)^{\frac{1}{p}} \)
(c) \( \sum_{n=1}^{\infty} |x_n y_n| \leq \left( \sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}} \left( \sum_{n=1}^{\infty} |y_n|^p \right)^{\frac{1}{q}} \)
(d) \( \sum_{n=1}^{\infty} |x_n + y_n| \leq \left( \sum_{n=1}^{\infty} |x_n|^p \right)^{\frac{1}{p}} + \left( \sum_{n=1}^{\infty} |y_n|^p \right)^{\frac{1}{q}} \)

58. A mapping \( f \) from a topological space \( X \) into a topological space \( Y \) is said to be continuous on \( X \) if:
(a) for every open set \( V \subset Y \), \( f^{-1}(V) \) is open in \( X \).
(b) for every open set \( V \subset Y \), \( f(V) \) is open in \( Y \).
(c) there exists an open set \( V \subset Y \) such that \( f^{-1}(V) \) is open in \( X \).
(d) there exists an open set \( V \subset X \) such that \( f(V) \) is open in \( Y \).
59. A complete inner product space is called
(a) Banach space
(b) Hilbert space
(c) normed linear space
(d) metric space

60. Which of the following is NOT true.
(a) Every Hilbert space can be made into a Banach space
(b) Every Banach space can be made into a Hilbert space
(c) Every complete inner product space is a Hilbert space
(d) Every complete normed linear space is a Banach space.

61. Which of the following is NOT a property of an inner product space.
(a) \( \langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle \)
(b) \( \langle xy, z \rangle = \langle x, z \rangle \langle y, z \rangle \)
(c) \( \alpha \langle x, z \rangle = \langle \alpha x, z \rangle \)
(d) \( \langle x, x \rangle \geq 0 \)

62. The matrix \( A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ell & 0 \\ 0 & 0 & x \end{bmatrix} \)
(a) Hermitian, skew Hermitian
(b) Hermitian, not skew Hermitian
(c) not Hermitian, skew Hermitian
(d) not Hermitian, not skew Hermitian

63. For any natural number \( n \): \( \lim_{n \to \infty} \frac{x^n}{x!} = 
(a) 0 
(b) 1 
(c) n 
(d) \infty 

64. Let \( G \) be an infinite cyclic group. Then \( G \) has
(a) At least two generators
(b) Almost two generators
(c) Exactly two generators
(d) Infinitely many generators

65. Let \( G = \{ 0, 1, 2, 3, 4, 5 \} \) be a group under addition modulo 6. Then the orders of the elements 2, 4, 5 are
(a) 2, 3, 6
(b) 3, 3, 6
(c) 4, 2, 1
(d) 3, 2, 6
66. Let \( R \) be a ring and \( S \) be an ideal in \( R \). Then \( S \) is said to be a prime ideal of \( R \) if
(a) \( ab = 0 \) implies either \( a = 0 \) or \( b = 0 \)
(b) \( ab \in R, a, b \in S \) implies either \( a \in R \) or \( b \in R \)
(c) \( ab \in S, a, b \in R \) implies either \( a \in S \) or \( b \in S \)
(d) every element of \( S \) is prime.

67. Let \( \{ a_n \}_{n=1}^\infty \) be the sequence \( \{1, 2, \frac{1}{2}, 3, \frac{1}{3}, 4, \frac{1}{4}, \ldots \} \). Then
(a) \( \lim_{n \to \infty} a_n = 0 \)
(b) \( \lim_{n \to \infty} a_n = \infty \)
(c) \( \lim_{n \to \infty} a_n = \{0, \infty\} \)
(d) \( \lim_{n \to \infty} a_n \) does not exist.

68. \( \lim_{n \to \infty} \frac{(3n+1)(n-2)}{n(n+3)} = \)
(a) 0
(b) 2
(c) 3
(d) \( \infty \).

69. The function \( f(x) = |x| + |x-1| \) is
(a) differentiable at 0 and 1
(b) not differentiable at 0 and 1
(c) is differentiable only in \( 0 < |x| < 1 \)
(d) is differentiable only in \( (0 < |x| < 1) \cup (|x| > 1) \)

70. \( \lim_{n \to \infty} \frac{\frac{1}{x^n}}{\frac{1}{x^n+1}} = \)
(a) 0
(b) 1
(c) \( \infty \)
(d) does not exist

71. All possible units of the integral domain of Gaussian integers are
(a) 1
(b) 1, -1
(c) \( i, -i \)
(d) 1, -1, \( i, -i \)

72. \( \lim_{x \to 0} \frac{x e^{x} - \log(1+x)}{x^2} = \)
(a) \( \frac{1}{2} \)
(b) \( \frac{1}{3} \)
(c) 0
(d) does not exist.
73. A bounded function $f$ is integrable on $[a, b]$ if and only if
(a) for every $\varepsilon > 0$, there exists a partition $P$ such that $U(P, f) - L(P, f) < \varepsilon$
(b) for every $\varepsilon > 0$, there exists a partition $P$ such that $L(P, f) - U(P, f) < \varepsilon$
(c) there exists $\varepsilon > 0$, and a partition $P$ such that $U(P, f) - L(P, f) < \varepsilon$
(d) there exists $\varepsilon > 0$, and a partition $P$ such that $L(P, f) - U(P, f) < \varepsilon$

74. In a $T_1$ topological space,
(a) Every singleton set is closed
(b) Every singleton set is open
(c) for any two distinct points, $x, y$, there exist disjoint open sets one containing $x$, other containing $y$
(d) for any two distinct points, $x, y$ there exist disjoint closed sets one containing $x$, other containing $y$

75. The integral equation $y(x) = 1 + \int_0^x (x - t) y(t) dt$ taking $y_0(x) = 1$ is solved by the method of successive approximation, then the solution is given by
(a) $y(x) = \cos x$
(b) $y(x) = \cosh x$
(c) $y(x) = \sinh x$
(d) $y(x) = e^x$

76. Using Euler's method with step size 0.1, the approximate value of $y(0.2)$ obtained for the initial value problem $\frac{dy}{dx} = x^2 - y^2$, $y(0) = 1$ is
(a) 1.122
(b) 0.820
(c) 0.980
(d) 0.890

77. The curve of quickest descent between the points $(x_1, y_1)$ and $(x_2, y_2)$ is a
(a) Cycloid
(b) Catenary
(c) Parabola
(d) Straight line.
78. Let \( x(t) = (x_1(t), x_2(t)) \) be the unique solution of the problem:
\[
\frac{d}{dt} x(t) = A x(t), \ t > 0, \ x(0) = (1, 1), \text{ where } A \text{ is real symmetric } 2 \times 2 \text{ matrix with }
\text{trace}(A) < 0 \text{ and } \text{det}(A) > 0. \text{ Then}
\]

(a) \( x_1(t) \to 0 \) and \( x_2(t) \to \infty \) as \( t \to \infty \)

(b) \( x_1(t) \to \infty \) and \( x_2(t) \to 0 \) as \( t \to \infty \)

(c) Both \( x_1(t) \) and \( x_2(t) \) tend to zero as \( t \to \infty \)

(d) Both \( x_1(t) \) and \( x_2(t) \) oscillate.

79. Consider the boundary value problem \( y'' + \lambda y = 0, y(0) = 0, y(\pi) = 0 \). Which of the following statements is correct?

(a) The eigenvalues of the above problem form a decreasing sequence of positive numbers \( \{\lambda_n\}_{n \in \mathbb{N}} \).

(b) The eigenfunctions of the above problem are orthogonal on the interval \([0, \pi/2]\).

(c) The sequence of the eigenvalues \( \{\lambda_n\}_{n \in \mathbb{N}} \) is bounded.

(d) The eigenvalues of the above problem form an increasing sequence of positive numbers \( \{\lambda_n\}_{n \in \mathbb{N}} \).

80. The subset of \( \mathbb{R}^2 \) in which the equation \( y_{xx} - 2y_{x} + x y_{xx} = 0 \) is of the hyperbolic type, is

(a) Compact and connected

(b) Connected but not compact

(c) Compact but not connected

(d) Neither connected nor compact.

81. **Buddhdev Dasgupta** is known as:

(a) a renowned athlete

(b) a renowned classical musician

(c) an eminent physicist

(d) an eminent bio-chemist

82. How many persons were awarded with Padma Bhushan award in 2012?

(a) 7

(b) 17

(c) 27

(d) 37

83. **Radio Broadcasting** began in India in ______

(a) 1917

(b) 1927

(c) 1937

(d) 1947
84. Army training Command is headquartered in Himachal Pradesh at?
   (a) Solan
   (b) Chamba
   (c) Hamirpur
   (d) Shimla

85. Creation of a new All India Civil Service is provided in which provision of the Constitution?
   (a) Article 311
   (b) Article 249
   (c) Article 201
   (d) Article 312

86. The Indian Diamond Institute is located at_______
   (a) Surat
   (b) Jaipur
   (c) Mumbai
   (d) Hyderabad

87. District Disaster Management Committee is headed by_______
   (a) The President / Chairman of the Zila Parishad
   (b) The Chief Executive Officer of the Zila Parishad
   (c) The Chairman of District Planning Committee
   (d) The District Collector

88. Who is the President of Ukraine?
   (a) Petro Poroshenko
   (b) Volodymyr Naumenko
   (c) Symon Petlyura
   (d) Stepan Vytvytskyi

89. Taj Mahal was built in_______
   (a) 1639
   (b) 1648
   (c) 1707
   (d) 1739

90. Rabindra Nath Tagore was awarded Noble prize for literature in which year?
   (a) 1913
   (b) 1915
   (c) 1919
   (d) 1920
1. Chaitrual festival is popular in ____
   (a) Sirmour Region
   (b) Kangra Region
   (c) Leh and Spiti
   (d) Tattapani Region

2. Which of the following districts in Himachal Pradesh has the highest number of crimes in 2013?
   (a) Kinnaur
   (b) Kangra
   (c) Kullu
   (d) Bilaspur

3. Samudayak Police Samiti is constituted in Himachal Pradesh at the level of ____
   (a) Beat Level
   (b) Sub-Division Level
   (c) Police Station Level
   (d) District Level

4. Which of the following lakes is located in Chamba District?
   (a) Bhrigir
   (b) Kumarwah
   (c) Kareri
   (d) Ghadasaru

5. Thapada is ______
   (a) Embroidered Shawal
   (b) Patchwork Quilt
   (c) Carpet
   (d) Wall hanging

6. Solang Nullah is famous for ______
   (a) Skiing Competition
   (b) Zorbing
   (c) Parachuting
   (d) All the above

7. Himachal Pradesh became a State on ______
   (a) 25th January, 1971
   (b) 26th January, 1971
   (c) 30th January, 1972
   (d) 25th January, 1973
98. Himachal Pradesh was made a part ‘C’ State in ____
   (a) 1948
   (b) 1950
   (c) 1951
   (d) 1956

99. The total area of the Hamirpur District is _______
   (a) 1230 Square K.M.
   (b) 1250 Square K.M.
   (c) 1118 Square K.M.
   (d) 1132 Square K.M.

100. Suket Satyagrah was led by _____________
    (a) Pandit Padam Dev
    (b) Surat Singh
    (c) Raja Lakshman Singh
    (d) Colonel G.S. Dhillon