TEST BOOKLET
AP (ASH) MATH-2016

Time Allowed : 2 Hours] [Maximum Marks : 100

All questions carry equal marks.

INSTRUCTIONS

1. Immediately after the commencement of the examination, you should check that test booklet does not have any unprinted or torn or missing pages or items, etc. If so, get it replaced by a complete test booklet.

2. Write your Roll Number only in the box provided alongside. Do not write anything else on the Test Booklet.

3. This Test Booklet contains 100 items (questions). Each item comprises four responses (answers). Choose only one response for each item which you consider the best.

4. After the candidate has read each item in the Test Booklet and decided which of the given responses is correct or the best, he has to mark the circle containing the letter of the selected response by blackening it completely with Black or Blue ball pen. In the following example, response “C” is so marked:

   A   B   C   D

5. Do the encoding carefully as given in the illustrations. While encoding your particulars or marking the answers on answer sheet, you should blacken the circle corresponding to the choice in full and no part of the circle should be left unfilled. After the response has been marked in the ANSWER SHEET, no erasing/liquid is allowed.

6. You have to mark all your responses ONLY on the ANSWER SHEET separately given according to ‘INSTRUCTIONS FOR CANDIDATES’ already supplied to you. Responses marked on the Test Booklet or in any paper other than the answer sheet shall not be examined.

7. All items carry equal marks. Attempt all items. Your total marks will depend only on the number of correct responses marked by you in the Answer Sheet. There will be no negative marking.

8. Before you proceed to mark responses in the Answer Sheet fill in the particulars in the front portion of the Answer Sheet as per the instructions sent to you.

9. If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct.

10. After you have completed the test, hand over the Answer Sheet only, to the Invigilator.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

P.T.O.
1. An example of uncountable set is:

(A) the set of all rational numbers in the interval \([0, 1]\)

(B) the set of all rational numbers in \(\mathbb{R}\)

(C) the set of all sequences whose elements are the digits 0 or 1

(D) the set \(\mathbb{Z}\) of all integers

2. The number of Hausdorff topology on a set with 5 elements is:

(A) 1

(B) 4

(C) 29

(D) 355

3. The Cantour set \(\mathbb{P}\) is not:

(A) compact

(B) perfect

(C) of positive Lebesgue measure

(D) uncountable
4. If \( \langle K_n \rangle \) is a sequence of non-empty compact set in a complete metric space \( X \), such that \( K_n \supseteq K_{n+1} \ (n = 1, 2, \ldots) \) and if
\[
\lim_{n \to \infty} \text{diam } K_n = 0,
\]
then:

\( (A) \quad \bigcap_{n=1}^{\infty} K_n = \emptyset \)

\( (B) \quad \bigcap_{n=1}^{\infty} K_n = \{x_0\} \text{ for some } x_0 \in X \)

\( (C) \quad \bigcup_{n=1}^{\infty} K_n = X \)

\( (D) \quad \bigcup_{n=1}^{\infty} K_n = \{x_0\} \text{ for some } x_0 \in X \)

5. If \( s_n = \frac{(-1)^n n}{n+1} \), then \( (\lim \sup_{n \to \infty} s_n, \lim \inf_{n \to \infty} s_n) \) equals:

\( (A) \quad (-1, 1) \)

\( (B) \quad (1, -1) \)

\( (C) \quad (0, 1) \)

\( (D) \quad (1, 0) \)

6. The partial sum \( S_{10} = \sum_{k=0}^{10} \frac{1}{k!} \) approximate the number e with error less than:

\( (A) \quad \frac{1}{10^{10}} \)

\( (B) \quad \frac{1}{10^{11}} \)

\( (C) \quad \frac{1}{10^{14}} \)

\( (D) \quad \frac{1}{10^7} \)
7. The series \( \sum_{n=1}^{\infty} a_n \) converges if:

(A) \( \limsup_{n \to \infty} (|a_n|)^{1/n} > 1 \)  
(B) \( \liminf_{n \to \infty} (|a_n|)^{1/n} < 1 \)

(C) \( \limsup_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} < 1 \)  
(D) \( \limsup_{n \to \infty} \frac{|a_n|}{|a_{n+1}|} < 1 \)

8. For a sequence \( \langle c_n \rangle \) of positive numbers:

(A) \( \liminf_{n \to \infty} \frac{c_{n+1}}{c_n} \leq \liminf_{n \to \infty} (c_n)^{1/n} \)

(B) \( \limsup_{n \to \infty} \frac{c_{n+1}}{c_n} > \liminf_{n \to \infty} (c_n)^{1/n} \)

(C) \( \limsup_{n \to \infty} \frac{c_{n+1}}{c_n} = \liminf_{n \to \infty} (c_n)^{1/n} \)

(D) \( \limsup_{n \to \infty} \frac{c_{n+1}}{c_n} > \limsup_{n \to \infty} (c_n)^{1/n} \)

9. Let \( m \) be the Lebesgue measure on \( \mathbb{R} \). Then, which one of the following is false?

(A) \( m(E) > 0 \) for non-empty open set

(B) \( m(E) = 0 \) if \( E \) is countable

(C) \( m(E) = 0 \) implies \( E \) is countable

(D) \( m(E) = 0 \) where \( E \subset [0, 1] \) consists of all numbers which possess decimal expansion not containing the digit 5
10. Let \( f(x) = |x| \). Then the upper right derivative and the upper left derivative of \( f \) at \( x = 0 \) are given by:

(A) \( D^+ = 1 = D^- \)  
(B) \( D^+ = 1, D^- = -1 \)  
(C) \( D^+ = -1, D^- = 1 \)  
(D) \( D^+ = 0 = D^- \)

11. If \( f : [0, 1] \to \mathbb{R} \) is given by \( f(x) = 0 \) when \( x \) is rational and \( f(x) = 1 \) when \( x \) is irrational, then:

(A) \( f \) is both Lebesgue integrable and Riemann integrable

(B) \( f \) is Lebesgue integrable but not Riemann integrable

(C) \( f \) is not Lebesgue integrable

(D) \( f \) is Riemann integrable

12. If \( f \) is a real-valued function and \( f^+, f^- \) are the positive and negative parts of \( f \) respectively, then:

(A) \( f = f^+ + f^- \)  
(B) \( f = f^+ - f^- \)  
(C) \( f = \max\{f^+, f^-\} \)  
(D) \( f + |f| = f^+ + f^- \)

13. A function analytic in \( G = \{z : \text{Re } z > 0\} \) is:

(A) \( \text{Re } z \)  
(B) \( \text{Im } z \)  
(C) \( \log z \)  
(D) \( \text{arg } z \)
14. The number of Möbius transformation having four fixed points is :

(A) 1  (B) 2

(C) 3  (D) 4

15. A branch of logarithm can be defined in the whole complex plane minus the set of points z on the real axis satisfying :

(A) $\text{Re } z \leq -1$  (B) $\text{Re } z > 1$

(C) $\text{Re } z > 0$  (D) $\text{Re } z > -1$

16. The Möbius transformation that maps, $0, 1, \infty$ to $1, 0, 2$ respectively is :

(A) $\frac{1-z}{2(1+z)}$  (B) $\frac{2(1-z)}{4+z}$

(C) $\frac{2(1-z)}{1+z}$  (D) $\frac{2(1-z)}{2-z}$

17. The value of the contour integral $\int_{|z|=1} \text{Re } z \, dz$ is :

(A) 0  (B) $\pi i$

(C) $2\pi i$  (D) $2\pi$
18. If \( \gamma(t) = a + re^{it}, \quad r > 0, \quad 0 \leq t \leq 6\pi, \) then the winding number of \( \gamma \) with respect to \( a \) is:

- (A) 0
- (B) 1
- (C) 2
- (D) 3

19. If \( \gamma_1(t) = 1 + e^{it}, \quad -\pi \leq t \leq \pi, \) \( \gamma_2(t) = 2 + 2e^{-it}, \quad -\pi \leq t \leq \pi \) and \( \gamma = \gamma_1 + \gamma_2, \) then the winding number \( n(\gamma, 3) \) is:

- (A) 0
- (B) 1
- (C) 2
- (D) −1

20. The value of the integral:

\[
\int_{|z|=4} \frac{5z^4 + 4z^3 + 3z^2 + 2z + 1}{z^5 + z^4 + z^3 + z^2 + z + 1} \, dz
\]

is:

- (A) 0
- (B) \( 2\pi i \)
- (C) \( 8\pi i \)
- (D) \( 10\pi i \)

21. If \( p \) is a polynomial of degree \( n \) and \( |p(z) - 1| > 1 \) for all \( z \) with \( |z| \geq R > 0 \), then \( \int_{|z|=R} \frac{p'(z)}{p(z)} \, dz \) equals:

- (A) 0
- (B) \( 2\pi i \)
- (C) \( n\pi i \)
- (D) \( 2n\pi i \)
22. Let $G = \{ z \in \mathbb{C} : 1 < |z| < 3 \}$ and $f : G \to \mathbb{C}$ be analytic. Then:

(A) $\int_{|z|=2} f(z) \, dz = 0$  \hspace{1cm} (B) $\int_{|z|=3} f(z) \, dz = 0$

(C) $\int_{|z-2|=1/2} f(z) \, dz = 0$  \hspace{1cm} (D) $\int_{|z-3|=3/2} f(z) \, dz = 0$

23. The singularity at $z = 0$ of the function $(\sin(z^2))/z$ is:

(A) removable \hspace{2cm} (B) pole

(C) non-isolated \hspace{2cm} (D) essential

24. The value of the integral:

$$\int_{|z|=1/2} \frac{\log(1-z)}{z^n} \, dz$$

is:

(A) $\frac{2\pi i}{n-1}$ \hspace{2cm} (B) $\frac{2\pi i}{n+1}$

(C) $\frac{2\pi i}{1-n}$ \hspace{2cm} (D) $2\pi n$

25. Let $w = (-1 + \sqrt{3})/2$ and $f(z) = 1/(z^3 - 1)$. The residue of $f(z)$ at $z = w$ is:

(A) $w/3$ \hspace{2cm} (B) $-w/3$

(C) 0 \hspace{2cm} (D) $w$
26. Let $\gamma : [0, 1] \to \mathbb{C}$ be given by $\gamma(t) = e^{4\pi it}$. The winding number of $\gamma$ with respect to 0 is:

- (A) 0
- (B) 1
- (C) 2
- (D) 4

27. The singularity of the function $1/{\sin(1/z)}$ at $z = 0$ is:

- (A) isolated
- (B) non-isolated
- (C) removable
- (D) essential

28. Let $D$ be a connected subset of $\mathbb{C}$. Then $D$ is simply-connected if:

- (A) $\mathbb{C}\setminus D$ is connected
- (B) $\mathbb{C}_\infty\setminus D$ is connected
- (C) $\int_{\gamma} f(z)dz = 0$ for every closed rectifiable curve and for some analytic function $f$
- (D) some non-vanishing analytic function has analytic square-root

29. The $K$-topology on $\mathbb{R}$ is the topology generated by the basis given by:

- (A) $B_1 = \{(a, b) | a, b \in \mathbb{R}\}$
- (B) $B_2 = \{[a, b] | a, b \in \mathbb{R}\}$
- (C) $B_4 = B_1 \cup \{(a, b)\setminus\{1/n : n = 1, 2, 3, \ldots\} | a, b \in \mathbb{R}\}$
- (D) $B_4 = B_2 \cup \{[a, b)\setminus\{1/n : n = 1, 2, 3, \ldots\} | a, b \in \mathbb{R}\}$
30. Let \( X = \{a, b, c\} \) with topology is \( \{\emptyset, X, \{a, b\}, \{b, c\}, \{b\}\} \). Then the sequence \( \langle x_n \rangle \) where \( x_n = b \):

(A) converges to \( b \) only  
(B) converges to \( a \) and \( c \) but not to \( b \)  
(C) converges to \( a, b, c \)  
(D) does not converge

31. The topologist's sine curve, which is the closure of the image of \( \sin(1/x) \), is:

(A) connected as well as path connected  
(B) connected but not path connected  
(C) path connected but not connected  
(D) neither connected nor path connected

32. A space that is not locally compact is:

(A) \( \mathbb{R} \)  
(B) \( \mathbb{Q} \)  
(C) \( \mathbb{R}^2 \)  
(D) \( [0, 1] \times [0, 1] \) in the dictionary order topology
33. Sorgenfrey plane is:
   (A) not regular
   (B) regular but not normal
   (C) normal
   (D) not Hausdorff

34. Let $X$ be a Hausdorff topological space. Urysohn lemma states that:
   (A) if each pair of disjoint open sets in $X$ can be separated by disjoint closed sets, then each such pair can be separated by continuous function
   (B) if each pair of disjoint open sets in $X$ can be separated by disjoint connected sets, then each such pair can be separated by continuous function
   (C) if each pair of disjoint closed sets in $X$ can be separated by disjoint open sets, then each such pair can be separated by continuous function
   (D) if each pair of disjoint closed sets in $X$ can be separated by disjoint compact sets, then each such pair can be separated by continuous function

35. Tietze extension theorem deals with the problem of extending:
   (A) a real-valued continuous function defined on closed subspace of a normal space $X$ to all of $X$
   (B) a real-valued continuous function defined on closed subspace of a regular space $X$ to all of $X$
   (C) an integer-valued continuous function defined on closed subspace of a normal space $X$ to a real-valued function on all of $X$
   (D) an integer-valued continuous function defined on closed subspace of a regular space $X$ a real-valued function on all of $X$
36. The space $l^p$ is not separable if:

(A) $p = 1$  
(B) $1 < p < 2$

(C) $1 \leq p < \infty$  
(D) $p = \infty$

37. Let $X$ be the set of all continuous functions on $[a, b]$. Then $X$ is complete in the metric $d$ where:

(A) $d(x, y) = \max_{a \leq t \leq b} |x(t) - y(t)|$

(B) $d(x, y) = \int_0^1 |x(t) - y(t)| \, dt$

(C) $d(x, y) = \left( \int_0^1 |x(t) - y(t)|^2 \, dt \right)^{1/2}$

(D) $d(x, y) = \left( \int_0^1 |x(t) - y(t)|^3 \, dt \right)^{1/3}$

38. The definition of normed space was given in 1922 by three mathematicians. The one who is not associated with it is:

(A) S. Banach  
(B) D. Hilbert

(C) H. Hahn  
(D) N. Wiener

39. The dual of the normed space $l^p$ is itself if:

(A) $p = 0$  
(B) $p = 1$

(C) $p = 2$  
(D) $p = \infty$
40. A linear operator that is not bounded is:

(A) zero operator from X to Y

(B) identity operator from X to Y

(C) Differential operator on the space of all polynomials on [0, 1] with norm given \( \|x\| = \max_{0 \leq t \leq 1} |x(t)| \)

(D) \( T : C[0, 1] \rightarrow C[0, 1] \) by \( y = Tx \) where \( y(t) = \int_{0}^{1} k(t, \tau) x(\tau) \, d\tau \) for a given continuous function on \([0, 1] \times [0, 1]\)

41. Which one of the following is not a property of a unitary operator \( U \) on a Hilbert space \( H \neq 0 \)?

(A) \( U \) is isometric

(B) \( U^* \) is unitary

(C) \( U^{-1} = U^* \)

(D) \( \|U\| < 1 \)

42. A space that is not reflexive is:

(A) a finite-dimensional normed linear space

(B) a Hilbert space

(C) the space \( l_1 \)

(D) the space \( l^p \) for \( 1 < p < \infty \)
43. If \( p(x) = 2x^2 - 3x + 4 \) and \( A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix} \), then \( p(A) \) equals:

(A) \( \begin{pmatrix} 9 & 6 \\ 4 & 13 \end{pmatrix} \)  
(B) \( \begin{pmatrix} 9 & 2 \\ 0 & 13 \end{pmatrix} \)  
(C) \( \begin{pmatrix} 9 & 0 \\ 2 & 13 \end{pmatrix} \)  
(D) \( \begin{pmatrix} 9 & 2 \\ 2 & 13 \end{pmatrix} \)

44. If \( A \) and \( B \) are invertible square matrices of the same order, which one of the following matrix is not necessarily invertible?

(A) \( A^T \)  
(B) \( A + B^T \)  
(C) \( AB^T \)  
(D) \( A^{-1}B \)

45. The value of the determinant:

\[
\begin{vmatrix}
0 & -1 & 1 & 3 \\
1 & 2 & -1 & 1 \\
0 & 0 & 3 & 3 \\
0 & 1 & 8 & 0
\end{vmatrix}
\]

is:

(A) 18  
(B) -18  
(C) 6  
(D) 5
46. If \( u = (1, 1, 2) \), \( v = (1, 0, 1) \) and \( w = (3, 1, 4) \) are vectors in \( \mathbb{R}^3 \), then \( \{u, v, w\} \):

(A) is linearly independent  
(B) is a basis for \( \mathbb{R}^3 \)

(C) does not span \( \mathbb{R}^3 \)  
(D) is an orthogonal basis for \( \mathbb{R}^3 \)

47. If \( A \) is a \( 3 \times 2 \) matrix of rank 1, then the dimension of null space of \( A^T \) is:

(A) 0  
(B) 2

(C) 3  
(D) 1

48. Let \( P_n \) be the vector space of all polynomials of degree less than or equal to \( n \). The matrix of the transformation \( T : P_1 \rightarrow P_2 \) given by \( T(p(x)) = xp(x) \) with respect to the standard bases is:

(A) \[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
1 & 1 \\
\end{bmatrix}
\]
49. The rank of the matrix:

\[
\begin{bmatrix}
1 & 1 & 1 & -1 \\
1 & 2 & 3 & 4 \\
3 & 4 & 5 & 2
\end{bmatrix}
\]

is:

(A) 1  
(B) 2

(C) 3  
(D) 4

50. A matrix that is diagonalizable is:

\[
\begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 4 & 7 \\
0 & 2 & 0 \\
0 & 0 & 3
\end{bmatrix}
\]

51. The characteristic polynomial of the matrix with respect to standard basis of \(\mathbf{R}^2\) of the rotation in \(\mathbf{R}^2\) counterclockwise by 90 degree is:

(A) \(\lambda^2 - 1\)  
(B) \(\lambda^2 + 1\)

(C) \(\lambda^2 + \lambda\)  
(D) \(\lambda^2 - \lambda\)
52. Well-ordering principle states that every non-empty set of positive:

(A) real numbers contain a smallest member

(B) integers contain a smallest member

(C) real numbers contain a largest member

(D) integers contain a largest member

53. A function that is neither one-to-one nor onto is:

(A) \( f : \mathbb{Z} \to \mathbb{Z}, f(x) = x^3 \)  
(B) \( f : \mathbb{R} \to \mathbb{R}, f(x) = x^3 \)

(C) \( f : \mathbb{Z} \to \mathbb{N}, f(x) = |x| \)  
(D) \( f : \mathbb{Z} \to \mathbb{Z}, f(x) = x^2 \)

54. The number of subgroups of \( \mathbb{Z}_{30} \), the group of integers 0, 1, 2, ..., 29 under addition modulo 30 is:

(A) 2  

(B) 3  

(C) 8  

(D) 30

55. If \( A \) is square matrix and \( A^3 = 0 \), then the inverse of \( I + A \):

(A) does not exist  

(B) equals \( I + A + A^2 \)

(C) equals \( I - A + A^2 \)  

(D) equals \( I + A - A^2 \)

56. If the order of an element \( a \) in a group is 30, then \( \langle a^3 \rangle \) equals:

(A) \( \langle a^6 \rangle \)  

(B) \( \langle a^{10} \rangle \)

(C) \( \langle a^{15} \rangle \)  

(D) \( \langle a^{21} \rangle \)
57. The special linear group $SL(2, \mathbb{R})$ of $2 \times 2$ matrices over $\mathbb{R}$ is the group under ................ of all matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfying the condition ................

(A) addition, $ad - bc \neq 0$  
(B) addition, $ad - bc = 1$

(C) multiplication, $ad - bc \neq 0$  
(D) multiplication, $ad - bc = 1$

58. The possible orders of elements in the symmetric group of degree 7 are:

(A) 1, 2, 3, ........, 7, 8, 9  
(B) 1, 2, 3, ........, 7, 9, 10

(C) 1, 2, 3, ........, 7, 8, 12  
(D) 1, 2, 3, ........, 7, 10, 12

59. Let $\mathbb{R}^\ast$ be the multiplicative group of non-zero real numbers and $\mathbb{R}$ be additive group of all real numbers. The mapping that is not a homomorphism is:

(A) $\phi : \mathbb{R}^\ast \to \mathbb{R}^\ast$, $\phi(x) = |x|$  
(B) $\phi : \mathbb{R}^\ast \to \mathbb{R}^\ast$, $\phi(x) = x^2$

(C) $\phi : \mathbb{R} \to \mathbb{R}^\ast$, $\phi(x) = x^2$  
(D) $\phi : \mathbb{R} \to \mathbb{R}^\ast$, $\phi(x) = e^x$

60. The maximal ideals in $\mathbb{Z}_{36}$, the ring of integers modulo 36 are:

(A) $\langle 2 \rangle$, $\langle 3 \rangle$  
(B) $\langle 2 \rangle$, $\langle 5 \rangle$

(C) $\langle 3 \rangle$, $\langle 5 \rangle$  
(D) $\langle 3 \rangle$, $\langle 18 \rangle$

61. Which one of the following is false?

(A) There is a field with four elements

(B) $\mathbb{Z}_4$ is a field

(C) Any two fields of order 4 are isomorphic

(D) $\mathbb{Z}_2$ is a field
62. An ideal $A$ of a commutative ring $R$ is maximal if:

(A) $a, b \in R, ab \in A$ implies $a \in A$ or $b \in A$

(B) $ab = 0$ implies $a = 0$ or $b = 0$

(C) $A \neq R, A \subseteq B \subseteq R, B$ an ideal of $R$ implies $A = B$ or $B = R$

(D) $a \in A, b \in A$ implies $ab \in A$

63. Eisenstein's criterion for a polynomial $a_0 + a_1x + \ldots + a_nx^n$ with integer coefficients to be irreducible over rational numbers is that there is a prime number $p$ such that:

(A) $p | a_0, p | a_1, \ldots, p | a_{n-1}, p | a_n, p^2 \nmid a_0$

(B) $p | a_0, p | a_1, \ldots, p | a_{n-1}, p | a_n, p \nmid a_0^2$

(C) $p \nmid a_0, p \nmid a_1, \ldots, p \nmid a_{n-1}, p | a_n, p^2 \nmid a_n$

(D) $p | a_0, p | a_1, \ldots, p | a_n, p^2 \nmid a_0$

64. The real root of the equation $x \sin x + \cos x = 0$ in $(2, 3)$ by using bisection method is:

(A) 2.796875

(B) 2.847313

(C) 2.98755

(D) 2.98756

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65. Newton-Raphson iteration formula for finding the cube root of a positive constant $c$ is:

(A) \( x_{n+1} = \frac{2x_n^3 + c}{2x_n^2} \)  
(B) \( x_{n+1} = \frac{3x_n^3 + c}{3x_n^2} \)

(C) \( x_{n+1} = \frac{2x_n^3 + c}{3x_n^2} \)  
(D) \( x_{n+1} = \frac{2x_n^3 - c}{3x_n^2} \)

66. Which one of the formulae is correct?

(A) \( V = 1 - E^{-1} \)  
(B) \( V = 1 + E^{-1} \)

(C) \( V = -1 + E^{-1} \)  
(D) \( V = -1 - E^{-1} \)

67. Simpson $1/3$ rule for evaluating the integral \( \int_a^b f(x)dx \) requires the interval \([a, b]\) to be divided into:

(A) an even number of subintervals of equal width

(B) any number of subintervals of equal width

(C) an odd number of subintervals of equal width

(D) any number of subintervals

68. Backward Euler method for solving the differential equation \( y' = f(x, y) \) is:

(A) \( y_{n+1} = y_{n} + hf(x_{n}, y_{n}) \)

(B) \( y_{n+1} = y_{n} + hf(x_{n} + 1, y_{n} + 1) \)

(C) \( y_{n+1} = y_{n} - 1 + 2hf(x_{n}, y_{n}) \)

(D) \( y_{n+1} = (1 + h)f(x_{n} + 1, y_{n} + 1) \)
69. Which one of the following is an elliptic partial differential equation?

(A) Laplace equation  (B) Wave equation

(C) Heat equation  (D) $u_{xx} + 2u_{xy} - 4u_{yy} = 0$

70. The two-dimensional wave equation is:

(A) $u_{tt} = c^2(u_{xx} + u_{yy})$  (B) $u_{xx} = c^2(u_{yy} + u_{zz})$

(C) $u_{zz} = c^2(u_{xx} + u_{yy})$  (D) $u_{tt} = c^2(u_{xx} - u_{yy})$

71. The general solution of the partial differential equation $z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$ is:

(A) $z = xyG(y/x)$  (B) $z = yG(x/y)$

(C) $z = xyG(x/y)$  (D) $z = G(y/x)$

72. The partial differential equation of the transverse vibrations of a string is:

(A) $u_{tt} = c^2u_{xx}$  (B) $u_{tt} = c^2u_x$

(C) $u_{tt} = c^2(u_x + u_t)$  (D) $u_{tt} = c^2(u_{xx} + u_t)$

73. Lipschitz's constant for the function $f(x, y) = 4x^2 + y^2$ on $|x| \leq 1$, $|y| \leq 1$ is:

(A) 1  (B) 2

(C) 3  (D) 4
74. The solution of the differential equation \( y' = (1 - x)/y \) represents:
   
   (A) a family of circles centered at (1, 0)
   
   (B) a family of circles centered at (0, 0)
   
   (C) a family of circles centered at (-1, 0)
   
   (D) a family of straight lines with slope -1

75. The initial value problem \( y' = 2y^{1/2}, y(0) = 0 \) has:
   
   (A) no solution
   
   (B) unique solution
   
   (C) solution exists but not uniquely
   
   (D) exactly three solutions

76. The solution of the differential equation \( (1 + y^2)dx = (\tan^{-1} y - x)dy \) is:
   
   (A) \( x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y} \)
   
   (B) \( y = \tan^{-1} x - 1 + ce^{-\tan^{-1} x} \)
   
   (C) \( x = \tan^{-1} y + ce^{-\tan^{-1} y} \)
   
   (D) \( y = \tan^{-1} x + ce^{-\tan^{-1} x} \)

77. Rayleigh-Ritz method is used to:
   
   (A) find maxima
   
   (B) find minima
   
   (C) solve boundary value problem
   
   (D) solve initial value problem
78. The general solution of \( y' + 2xy = 2e^{-x^2} \) is:

(A) \( y = (2x + c)e^{-x^2} \)  \hspace{1cm} (B) \( y = 2(x^2 + c)e^{-x} \)

(C) \( y = e^{-x^2} + c \)  \hspace{1cm} (D) \( y = x^2e^{-x} + c \)

79. Surface whose tangent planes cut-off an intercept of constant length \( k \) from the axis of \( z \) is:

(A) \( f(x/y, (x - k)/y) = 0 \)  \hspace{1cm} (B) \( f(y/x, (z - k)/y) = 0 \)

(C) \( f(y/x, (z - k)/x) = 0 \)  \hspace{1cm} (D) \( f(x/y, (z - k)/y) = 0 \)

80. While solving \( y'' + 4y = \tan 2x \) by the method of variation of parameters, the value of Wronskian is:

(A) 1  \hspace{1cm} (B) 2

(C) 3  \hspace{1cm} (D) 4

81. What was the growth rate in the economy of Himachal Pradesh during 2013-14?

(A) 5.5%  \hspace{1cm} (B) 5.9%

(C) 6.2%  \hspace{1cm} (D) 6.7%
82. What was the total fruit production in H.P. during 2014-15 (upto December 2014, in Lakh Metric tons) ?

(A) 4.83  
(B) 5.76  
(C) 6.29  
(D) 6.53

83. According to Economic Survey 2014-15, what is the amount of old age pension per month in Himachal Pradesh (in rupees) ?

(A) 5,000  
(B) 4,000  
(C) 2,000  
(D) 1,000

84. When was Pradhan Mantri Jan Dhan Yojna launched in H.P. ?

(A) 26 January, 2014  
(B) 15 August, 2014  
(C) 28 August, 2014  
(D) 02 October, 2014

85. When did H.P. Vidhan Sabha unanimously pass a resolution demanding full statehood for the Pradesh ?

(A) March 1967  
(B) January 1968  
(C) October 1969  
(D) March 1970
86. What is Haar marriage (which is/was prevalent in some parts of H.P.)?
(A) When boy’s family selects the bride
(B) When girl’s family selects the boy
(C) When marriage is fixed after obtaining girl’s consent
(D) When the girl is kidnapped or elopes with the boy

87. Around which year did Jai Singh Kanheya defeat Jassa Singh Ramgarhia and took possession of Kangra?
(A) 1770 A.D.        (B) 1775 A.D.
(C) 1780 A.D.        (D) 1785 A.D.

88. Who was the President of All India State Peoples’ Conference around 1939 A.D.?
(A) Pattabhisitaramaiyya        (B) Dr. Rajendra Prasad
(C) Sardar Patel          (D) Jawaharlal Nehru

89. In which region of H.P. is Shiv Gufa (Savaur Village)?
(A) Bharmaur (Chamba)        (B) Karsog (Mandi)
(C) Rajgarh (Sirmaur)       (D) Gagret (Una)

90. Around which year was Bhoomi Bandobust Abhiyan launched in Bilaspur princely state?
(A) 1884 A.D.  (B) 1890 A.D.
(C) 1930 A.D.  (D) 1932 A.D.
91. In the Atal Pension Yojna for how many years premium has to be paid to become eligible for pension?

(A) Five years  
(B) Ten years  
(C) Fifteen years  
(D) Twenty years

92. Who is Maulana Arshad Madni?

(A) Chief of Hizbul Mujahideen  
(B) President of All India Jamiat Ulema-i-Hind  
(C) Top Taliban official in Pakistan  
(D) Bangladeshi leader tried for crimes against humanity during the 1971 liberation war

93. When did the Union Cabinet grant the status of national minority to Jains in India?

(A) 2010 A.D.  
(B) 2012 A.D.  
(C) 2013 A.D.  
(D) 2014 A.D.

94. Which bank was given permission to set up in 2014 A.D. in private sector?

(A) HDFC  
(B) IDFC  
(C) IDBI  
(D) None of these

95. On which date of Gregorian calendar (in a non-leap year) does the first day of Chaitra of the Saka calendar fall?

(A) February 20  
(B) March 22  
(C) April 13  
(D) April 14
96. With which of the following is Craig Venter associated?

(A) Radar  (B) Human Genome

(C) World Wide Web  (D) Hotmail

97. What is the currency of Myanmar?

(A) Kyat  (B) Rupee

(C) Rulyaa  (D) Ringgit

98. What is the name of the device fitted in a car exhaust to reduce pollution?

(A) Kitkat  (B) Catalytic converter

(C) Instagram  (D) Flicker

99. When was the League of Nations set up?

(A) 1919 A.D.  (B) 1920 A.D.

(C) 1921 A.D.  (D) 1922 A.D.

100. Which was the first newspaper published in English?

(A) London Times  (B) Washington Times

(C) Statesman  (D) Oxford Gazette

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