This question paper contains 4 printed pages]

HPAS (Main)—2012

STATISTICS

Paper I

Time : 3 Hours *Maximum Marks : 150*

Note :— Attempt Question No. 1 which is compulsory and any other *four* questions from the rest, *five* in all. *All* questions carry equal marks.

1. (*a*) A biased coin, for which the probability of head is 0.4, is tossed three times. Find the probability of getting at least one head.

(*b*) Two variables X and Y have correlation coefficient 'r'. Obtain the correlation coefficient between $U = (X - a)/c$ and $V = (Y - b)/d$ where a, b, c and d are constants.

(*c*) Explain the fitting of a second degree curve by the method of least squares.

P.T.O.

2. (*a*) For two events A and B, let $P(A) = 2/3$, $P(\bar{B}) = 1/2$ and $P(A \cap B) = 1/6$. Obtain the value of $P(A / A \cup B)$.

(*b*) A random variable X has the probability distribution given by p.d.f. :

$$f(x) = \begin{cases} 2x & ; \quad 0 \le x \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Obtain the value of E $3X(X - 1)$.

3. (*a*) Define a binomial distribution. Obtain its moment generating function and, from this, obtain its mean and variance.

(*b*) Define a normal distribution and state its important properties.

4. (*a*) Compare the relative merits of the mean, median and mode of a frequency distribution. Define the two 'ogives'.

(*b*) Prove that the mean deviation is least about its median.

5. (*a*) Find the mean of X and Y and the correlation coefficient between them from the following regression equations :

$$x - 2y + 50 = 0$$

$$3y - 2x - 10 = 0.$$

(*b*) Define the partial and multiple correlation coefficients. If $r_{12} = r_{23} = r_{31} = \rho$, then find the values of $r_{12.3}$ and $R_{1.31}$.

6. (*a*) Define a random sample. Explain the concepts of sampling distribution and its standard error.

(*b*) Define a χ^2-distribution. Show that the sum of two independent χ^2-statistics have a χ^2-distribution.

7. (*a*) When is an estimator said to be 'consistent' and 'unbiased' ? Obtain an unbiased estimator of σ^2 in N $(0, \sigma^2)$ based on a sample of size n.

(*b*) Define the 'sufficiency' of a statistic. Show that the sample mean $\bar{x}$ is sufficient for μ in the normal distribution $N(\mu, 1)$.

8. (*a*) What are maximum likelihood estimators ? Obtain the maximum likelihood estimator of standard deviation σ in a normal distribution :

$$f(x; \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2} \; (-\infty < x < \infty).$$

(*b*) Explain the concept of confidence interval, giving example.