This question paper contains 4 printed pages]

HPAS (Main)—2013

STATISTICS

Paper I

Time : 3 Hours *Maximum Marks* : 150

Note :— Attempt Question Number 1 which is compulsory and any other *four* questions from the rest, *five* in all. *All* questions carry equal marks. Symbols have their usual meanings.

1. (*a*) State whether the following are discrete or continuous random variables :

   (*i*) Measurement of temperature at a place over the number of months.

   (*ii*) Number of misprints on the pages of a book.

   (*iii*) Number of trees in a village.

   (*iv*) Number of experiments given in a practical class.

(*b*) In five tosses of a biased coin with Pr(H) = 2Pr(T), let X be the number of heads obtained. Calculate E(X).

(*c*) Mean and variance of a random variable are 8 and 12 respectively. Can it be a binomial variable or not ? Explain with reasons.

2. (*a*) State and prove Bayes Theorem.

(*b*) Let X and Y have joint probability density function :

$$f(x, y) = \begin{cases} x^2 + \frac{1}{3}xy\,; & \begin{matrix} 0 < x < 1 \\ 0 < y < 1 \end{matrix} \\ 0 & \text{elsewhere} \end{cases}$$

Determine :

$$\Pr\left(Y < \frac{1}{2} \Big/ X < \frac{1}{2}\right).$$

3. (*a*) Define rank correlation coefficient and establish an expression which measures it. Show that it lies between −1 and +1.

(*b*) What do you mean by dispersion ? Discuss various measures of dispersion.

4. (*a*) Define negative binomial distribution. Find its m.g.f. and hence its mean and variance.

(*b*) Obtain recurrence relation for finding moments of Poisson distribution.

5. (*a*) Define bivariate normal distribution and find its marginal and conditional distributions.

(*b*) Explain the method of least squares. Derive least square equations for fitting the curve :

$$y = ax + \frac{b}{x}.$$

6. (*a*) Explain the following with examples :

(*i*) Intraclass correlation

(*ii*) Partial correlation.

(*b*) Write a short note on interval estimation.

7. Let X_1, X_2, X_3 and X_4 are mutually independent normal variables with mean zero and variance unity. Find out the distributions of :

(i) $\frac{1}{2}(X_1 - X_2)^2 + X_3^2 + X_4^2$

(ii) $\frac{X_1}{X_2}$

(iii) $\frac{\sqrt{3}\, X_1}{\left(X_2^2 + X_3^2 + X_4^2\right)^{1/2}}$.

8. (a) State and prove Cramer-Rao Inequality.

(b) Find out the maximum likelihood estimation of θ in the following distribution :

$$f(x, \theta) = \begin{cases} 1 & \text{if } \theta - \frac{1}{2} \le x \le \theta + \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$