Note:— Attempt Question Number 1 which is compulsory and any other four questions from the rest, five in all. All questions carry equal marks. Symbols used in questions have their usual meanings.

1. (a) Explain the following with examples:
   
   (i) Null and Alternative hypotheses
   
   (ii) Median Test.
   
   (iii) Errors in testing of hypotheses
   
   (iv) Analysis of variance and two-way classification.
(b) Decide, in each of the following cases, whether the hypothesis is simple or composite:

(i) the hypothesis that a random variable has a normal distribution with mean $\mu = 50$.

(ii) the hypothesis that a random variable has a normal distribution with mean $\mu = 50$ and s.d. $\sigma > 5$.

(iii) the hypothesis that a random variable has an exponential distribution.

(iv) the hypothesis that a random variable has a Poisson distribution with $\lambda < 1.2$.

2. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size $n$ drawn from $N(\mu, \sigma^2)$; $\sigma^2$ is not known. Obtain the likelihood ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. 
(b) Describe sequential probability ratio test (SPRT) for testing a simple hypothesis against a simple alternative.

3. Let \( x_1, x_2, \ldots, x_n \) be a random sample from the rectangular population with density:

\[
f(x) =
\begin{cases}
\frac{1}{\theta}, & 0 \leq x \leq \theta \\
0, & \text{otherwise}
\end{cases}
\]

Consider the critical region \( x_{(n)} > 0.8 \) for testing the hypothesis \( H_0 : \theta = 1 \) where \( x_{(n)} \) is the maximum of sample values. What is the associated probability of type I error and what is the power function? Give detailed calculations.

4. Describe linear statistical models with examples. Find out OLSE of \( \beta \) in general linear model:

\[
Y = X\beta + \epsilon
\]
describing the assumptions usually made in this model.

Show that this estimator is unbiased. Also find out its
dispersion matrix.

5. (a) What are the basic principles of a sample
survey? Distinguish between sampling and
non-sampling errors.

(b) Explain the different sources of sampling and
non-sampling errors.

6. (a) Describe ratio method of estimation. Obtain the mean
and mean squared error of ratio estimator to the
first order of approximation.

(b) If the coefficient of variation of $x$ and $y$ are equal
and the correlation coefficient between them is $\frac{2}{3}$,
compute the efficiency of ratio estimator relative
to the sample mean calculated on the basis of
SRSWOR.
7. (a) Describe complete analysis of one-way classified data along with its model.

(b) What do you mean by design of experiments? Explain principles of experimental design.

8. Define factorial experiments. Describe, in detail the analysis of $2^3$ factorial experiment conducted in a randomised block design.