This question paper contains 4+1 printed pages]

CODE : FRO-2017

MATHEMATICS

Roll No. ....

Time : 3 *Hours* *Maximum Marks* : 200

Note :— (*i*) Question paper consists of two parts viz. Part I and Part II. Each part contains four questions. The paper as a whole carries eight questions. Question Nos. **1** and **5** are compulsory. The candidates are required to attempt *three* more questions out of the remaining six questions taking at least *one* question from each part i.e. this is in addition to the compulsory question of each part. Attempt *five* questions in all. All questions carry equal marks. The parts of a question are to be attempted at one place in continuation. Answers should be brief and to the point.

(*ii*) Parts of same question must be attempted together and not to be attempted in between the answers to other questions.

Part-I

1. Write short notes on the following :

   (*a*) Re-arrangements of series 8

   (*b*) Cayley Hamilton's Theorem. 8

   (*c*) Singular solution of a differential equation. 8

   (*d*) Taylor's theorem for function of a real variable. 8

   (*e*) Central Conicoids. 8

2. (*a*) Prove that every subgroup of a cyclic group is cyclic. 15

   (*b*) Solve the cubic $x^3 - 18x - 35 = 0$ by Cardan's method. 15

   (*c*) For any positive integer n, prove that $\sqrt{n}$ is either an integer or irrational number. 10

3. (*a*) State and prove ratio test for infinite series of positive terms. 15

   (*b*) State and prove the Lagrange mean value theorem and explain its geometrical interpretation. 15

(c) Find the equation of cone with vertex (5, 4, 3) and whose base is conic :

$3y^2 + 4z^2 = 16, \ 2x + z = 0.$ 10

4. (a) Solve the initial value problem : 15

$xy'' - y' = x^2 e^x, \ (x > 0)$

$y(1) = 0, \ y'(1) = 2.$

(b) Obtain the differential equation of the family of parabolas with axes parallel to the x-axis. Also, write the order and degree of the resulting equation. 15

(c) State and prove the fundamental theorem of integral calculus. 10

Part-II

5. Write short notes on the following :

(a) Gauss divergence theorem 8

(*b*) Degrees of freedom and constraints 8

(*c*) Compound pendulum and its time period 8

(*d*) Principle of virtual work 8

(*e*) Hydrostatic thrust on a curved surface. 8

6. (*a*) Explain the theme of Stokes theorem and write down its complete statement. Verify the theorem for the vector point function $\overline{\mathbf{F}} = -y^3\hat{i} + x^3\hat{j}$, where S is the circular disc $x^2 + y^2 \leq 1$, $z = 0$. 20

(*b*) A solid hemisphere of density ρ and radius a floats with its plane base immersed in a liquid of density ρ_ℓ $(\rho_\ell > \rho)$. Prove that the equilibrium is stable and find the metacentric height. 20

7. (*a*) Find the expression for potential of a thin uniform rod at an external point and hence show that equipotential curves are ellipses. 15

(*b*) Find the moment of inertia of the area bounded by curve $r^2 = a^2 \cos 2\theta$ about its axis. 15

(c) Obtain the Cartesian equation of a common catenary. 10

8. (a) A particle slides down a smooth cycloid starting from rest at the cusp, the axis being verical and vertex downwards. Prove that the magnitude of the acceleration of the particle is equal to g at every point of its path and that when it arrives at the vertex the pressure on the curve is equal to twice the weight of the particle. 20

(b) Explain the stability of equilibrium for unconstrained submerged bodies in a liquid by giving examples. 20