

This question paper contains 8 printed pages]

CODE : FS-17

MATHEMATICS

Time : 3 Hours

Maximum Marks : 200

- Note* :— (i) Attempt any *five* questions including question numbers 1 and 5, which are compulsory. Minimum of *two* questions including compulsory shall be attempted from each part. Be brief and to the point. Parts of the same questions must be answered together and not to be attempted in between the answers to other questions.
- (ii) Parts of same questions must be attempted together and not to be attempted in between the answers to other questions.

P.T.O.

Part I

1. (a) Define equivalence relation and test the relation of \sim for equivalence. 8
- (b) Using examples, illustrate the injective, surjective and bijective mappings. 8
- (c) Show that the determinant of an odd order skew-symmetric matrix always vanishes. 8
- (d) Explain, how can a set of non-zero rational numbers form a group ? 8
- (e) State and prove the fundamental theorem of arithmetic. 8
2. (a) Explain the working of Cauchy's test for convergence of infinite series and use it to check the convergence of :

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

for real p .

- (b) Express $\cos 7\theta$ in terms of powers of $\cos \theta$. 12
- (c) Explain, Cardan's method of solving a cubic equation. 8
- (d) Find the nature and location of the roots of the quartic equation : 8

$$x^4 + 6x^3 + 9x^2 - 4 = 0.$$

3. (a) Find the slope of a line, which is making an angle of $\pi/4$ with another line of slope $\frac{1}{2}$. 8
- (b) Find the equation of the hyperbola whose foci are $(0, \pm 12)$ and length of latus rectum is 36. 8
- (c) Derive the equation of normal at point ' t ' to the parabola $y^2 = 4ax$, considering the parametric representation. 8

(d) State Lagrange's mean value theorem and test its validity for the exponential function (e^x). 10

(e) Find the maximum value of the function : 6

$$f(x) = \sin x + \cos x .$$

4. (a) Show that area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. 8

(b) Find the complete solution of : 12

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x .$$

(c) Solve : 10

$$\frac{dy}{dx} = \frac{3y + 2x + 4}{4x + 6y + 5} .$$

(d) Solve :

$$(1 + x^2)dy = (\tan^{-1} x - y)dx ,$$

for y being a function of x . 10

Part II

5. (a) Show that the angle between unit vectors \hat{A} and \hat{B} is given by : 8

$$2\sin^{-1}\left(\frac{1}{2}|\hat{A} - \hat{B}|\right).$$

- (b) Write the physical interpretation of the divergence of a vector point function. 8
- (c) Show that : 8

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}).$$

- (d) Show that :

$$\nabla^2\left(\frac{1}{r}\right) = 0,$$

for r being the magnitude of position vector. 8

- (e) Verify Stokes' theorem for the vector field :

$$F(x, y, z) = yz^2\hat{x} + zx^2\hat{y} + xy^2\hat{z}$$

taken around the rectangle made of the lines

$$x = \pm a, \quad y = 0, \quad y = b. \quad 8$$

6. (a) What are the *six* fundamental principles, which form the basis for study of statics ? 6

- (b) Describe the principle of virtual work and derive the Lagrange's equations for a system of N point particles. 18

- (c) Discuss the laws of friction. 8

- (d) Position of moving particle in (x, y) plane at time t is given by :

$$x = 2t^3, \quad y = t^2 + 1.$$

Determine the equation of path of particle, its

velocity and acceleration. 8

7. (a) A particle is projected at angle α with initial velocity u . Find the velocity and angle of motion of projectile after a given time t . 8
- (b) Find the velocity of B relative to A, when A is moving towards north with velocity u and B is moving α radians east of north with velocity v . 8
- (c) Show that, of a particle along a circular path, angular velocity is the ratio of its linear velocity to radius of the circle. 8
- (d) Derive expression for period of simple pendulum of given length of string. 8
- (e) What is the principle of work and energy ? 8

8. Write short notes on the following :

- (a) Centre of pressure 8
- (b) Sufficient condition for stability 8
- (c) Convective equilibrium 8
- (d) Surface of floatation 8
- (e) Isothermal atmosphere. 8